Transfers versus Public Investment: The Politics of Intergenerational Redistribution and Growth

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Abstract

In this paper we analyze tax and transfer choices in an OLG economy with capital accumulation and endogenous growth coming from public investment, such as education. We solve for a Markov perfect equilibrium when electoral competition targets the votes of young and old households. We find that when calibrating the model to match US data, it predicts levels of intergenerational transfers and of public investments that are similar to the observed ones. Furthermore the Ramsey policy for the same parameters would call for both generations to be taxed to finance public investment. If the political process internalized the benefits that public investment has on future generations, growth would be twice as high as currently observed.

JEL Classification Code: E62, H55, O41

1 Introduction

In developed countries, intergenerational transfers are much larger than intragenerational ones. Data from the OECD shows that for every dollar of government spending spent on transfers for middle-aged individuals, 62 cents are spent per child aged $0-14$, and 2.85 dollars are spent on the old. Thus the ratio of intergenerational transfers to intragenerational ones is almost 3.5.[1] Within developed nations, there are substantial differences in the relative sizes of these transfers. The UK destines 2.13 pounds to the elderly for every pound spent on the middle

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[1]To complete the comparison we need to include public investment in infrastructure decomposing its contemporaneous and long-term impact. According to the literature on the impact of infrastructure on growth, the long-term impact is more important.

aged, and Italy spends slightly more on per child than per middle aged. The ratio of spending per elderly individual to spending per child varies from 2.3 in Japan and Sweden to 3.8 in Italy and the US.

We know that the level of public investment in infrastructure and public education has a significant impact on a countrys long-run growth performance.[2] Therefore it seems a relevant task to explore the determinants of the wide disparities in policy choice, and growth performance, observed across countries. One way to proceed is to assume that policies are chosen collectively according to some aggregation of preferences in society. If this is the case, then observed policy differences can depend on differences in economic primitives, population characteristics and on the details of the policy selection processes.

In this paper we set up a model that captures how population growth affects political choices and therefore growth. This is contrary to traditional models of endogenous growth (See Barro Sala-i-Martin (1995) chapter 4) were population growth is found to have no effect on long-run growth. By considering finite horizons we introduce a tension between the middle aged and the old about the value of public investments. The former might want to finance education because they would live enough to reap part of the benefits in the form of higher return to their savings. But the latter receive little economic benefit from public investments and are better off supporting the introduction of direct transfers to them.

We introduce political choice of public investments and social security transfers in the Diamond (1965) overlapping-generations framework with production and capital accumulation with endogenous growth. Households are assumed to be non-altruistic. As consumers, they are price takers. As voters, they rationally take into account how policies affect prices and future political choices; furthermore, they are not bound by past political decisions. The politico-economic equilibrium therefore features subgame-perfect tax and transfer choices that support a competitive equilibrium.

We model electoral competition under the assumption of probabilistic voting rather than a pivotal median voter. In equilibrium, vote-maximizing candidates aim at maximizing the average welfare of all voters, not only of the median voter. We restrict attention to Markov perfect equilibria where policy choices are only a function of the natural state variables. Political competition results in lower public investment than what is optimal according to the Ramsey

[2] See Barro (1990) for a first treatment, and Gerhard Glomm and B. Ravikumar 1997, JEDC, 21, , 183–204 for an OLG setting.

plan. The political process does not fully internalize the positive impact of public investment and therefore "wastes" public resources as transfers to the old.

2 Literature

2.1 Economics only, Public Investment in Endogenous Growth Models, Optimal Investment/Transfer Combination

Boldrin Montes (2005) show that in order to replicate a complete markets allocation when capital markets are incomplete there is a need to use both social security and public education, linking them in a certain way. This is one example of a new literature that tries to find economic and political reasons for the joint existence of social security and education. In any case it is not clear why the complete markets allocation is the relevant benchmark, given the education introduces a positive externality in the economy.

2.2 Politics, Intra-Generational Redistribution through Public Education

Gerhard Glomm and B. Ravikumar 1992, JPE, 100, 4, 818–834. This is a simple model in which parents vote on whether public education should be provided and if so, at what tax rate, or whether education should be privately provided by each household. This trade-off is affected by parents' ability to bequeath human capital. Public education reduces income inequality, but leads to lower long-run growth. There is no alternative policy instrument, and no intergenerational conflict in this setup.

2.3 Politics, Inter-Generational Redistribution and Education/Investment

Bellettini Berti Ceroni (1999). The authors argue that although redistributive and growth-oriented policies compete for scarce tax revenues, they might go hand in hand since the former are needed to make growth socially palatable. Growth is driven by accumulation of public capital; therefore more redistribution depresses growth. Without a link between current and past policy choices (static Nash) zero taxes are chosen. But if public capital is sufficiently productive subgame-perfect Nash equilibria based on trigger strategies (interpreted as a social norm) can be supported. If SS is expected to be sustained tomorrow then young also want to invest in public capital since this increases future wages. The setup is similar to the one we develop in this paper. By adding probabilistic voting we increase the resistance against public investments, and we eliminate trigger strategy equilibria.

Rangel (2003). Model of intergenerational (IG) exchange to study the conditions under which nonmarket institutions can generate Pareto-optimal levels of investment. Agents live for three periods. Middle-aged decide on investment in forward IG good (FIG) that benefits future generations only. If the only decision made every period is how much to invest in FIGs, no investment takes place. But a link to the provision of backward IG goods (BIG, for example, the government transfers resources to the elderly through the social security system) can sustain the provision of FIGs. Without backward exchange, investment in FIGs is inefficiently low; but with it, even optimal investment by selfish generations is possible. Linkages across games thus play an important role in sustaining cooperation, as realized before in the literature on multimarket contact in industrial organization. *If* a majority of the electorate receives positive benefits from keeping the social security system, there are voting equilibria in which even selfish generations vote to invest in FIGs. In these equilibria, investment in future generations is supported by a link between BIGs and FIGs: present voters correctly believe that future voters support of social security depends on whether or not they invest in FIGs. This is another paper in the above mentioned new literature on linkages between social security and education, stressing the political side now.

2.4 Other

Poterba 1997/8: Using a panel of state level data in the US he shows that districts with more elderly go hand in hand with less education spending.

3 The Model

We consider an economy inhabited by overlapping generations of (continua of) consumer-voters that are economically and politically active for two periods, as workers (when young) and retirees (when old). The gross population growth rate and thus, the ratio of young to old households, equals ν. In each period, workers and retirees elect a government that runs an intergenerational transfer scheme and undertakes public investment.

3.1 Production

A continuum of competitive firms transform capital and labor into output by means of a Cobb-Douglas technology with time-varying productivity. Output per retiree in period t is given by

$$B_0 A_t^{1-\alpha} s_{t-1}^{\alpha} [\nu(1-x_t)]^{1-\alpha},$$

where $B_0 > 0$ and the capital share $\alpha \in (0,1)$. Capital is owned by retirees and fully depreciates after one period. The capital stock per retiree, s_{t-1}, therefore equals the per-capita savings of workers in the previous period. Labor is supplied by current workers. Normalizing their time-endowment to unity and denoting workers' leisure consumption by x_t, labor supply per retiree equals $\nu(1-x_t)$. Note that we assume the exponents on labor input and the time-varying component of productivity, A_t, to be the same. On a balanced-growth path, the ratio A_t/s_{t-1} will therefore be constant.

Production factors are rewarded according to their marginal products, due to competition among firms. The wage, w_t, and the gross return on private capital, R_t, therefore satisfy

$$\begin{aligned} w_t &= (1-\alpha) B_0 A_t^{1-\alpha} s_{t-1}^{\alpha} [\nu(1-x_t)]^{-\alpha}, \\ R_t &= \alpha B_0 A_t^{1-\alpha} s_{t-1}^{\alpha-1} [\nu(1-x_t)]^{1-\alpha} = w_t \frac{\nu(1-x_t)}{s_{t-1}} \alpha' \end{aligned}$$

with $\alpha' \equiv \alpha/(1-\alpha)$.

Productivity is endogenous, reflecting public investments during previous periods. More specifically, we assume productivity growth to be a concave function of lagged public investment relative to previous period's productivity level,

$$A_{t+1} = B_1 A_t^{1-\delta} I_t^{\delta}$$

with $B_1 > 0$, $\delta \in (0,1)$, and I_t denoting public investment per retiree. The link between public investment and productivity growth can be interpreted in several ways. According to our preferred interpretation, I_t and A_t represent publicly provided education and "human capital", respectively. According to this interpretation, households live for three periods although they are economically and politically active only during the last two. As (very young) students, households enjoy public education but do not consume nor work nor vote. In the following period, as (young) workers, households contribute with their human capital both to production and the formation of new human capital for the succeeding cohort.[3] According to an alternative

[3]This interpretation in terms of efficiency enhancing human capital becomes clearer by rewriting output per retiree as

$$B_0 s_{t-1}^{\alpha} [\nu(1-x_t) A_t]^{1-\alpha}.$$

interpretation, I_t and A_t represent (investments into) public infrastructure.

3.2 Government

The government taxes labor income in period t at rate $\tau_t + \sigma_t + \xi_t$ and capital income at rate $\eta_t + \theta_t$. Revenues collected from workers fund transfers to retirees (the component corresponding to τ_t), public investment (σ_t), as well as a lump-sum rebate to workers (ξ_t). The only role of ξ_t therefore is to distort labor supply. Revenues collected from retirees fund transfers to workers (the component corresponding to η_t) as well as public investment (θ_t). Denoting per-capita transfers to workers and retirees by a_t and b_t, respectively, we then have

$$\begin{aligned} a_t &= w_t(1-x_t)\xi_t + s_{t-1}R_t\eta_t/\nu = w_t(1-x_t)(\xi_t + \eta_t\alpha'), \\ b_t &= \nu w_t(1-x_t)\tau_t, \\ I_t &= \nu w_t(1-x_t)\sigma_t + s_{t-1}R_t\theta_t = \nu w_t(1-x_t)(\sigma_t + \theta_t\alpha'). \end{aligned}$$

Tax rates must not exceed unity nor be negative (since we exclude lump-sum taxes): $0 \leq \tau_t + \sigma_t + \xi_t \leq 1$ and $0 \leq \eta_t + \theta_t \leq 1$ for all t. Further constraints follow from the requirement that a_t, b_t, and I_t are positive. Taken together, these restrictions imply that the policy instruments have to satisfy the following conditions:

$$\begin{array}{l} \xi_t + \eta_t\alpha' \geq 0, \quad \tau_t \geq 0, \quad \sigma_t + \theta_t\alpha' \geq 0, \\ 1 \geq \tau_t + \sigma_t + \xi_t \geq 0, \quad 1 \geq \eta_t + \theta_t \geq 0, \end{array} \quad \text{for all } t. \tag{1}$$

We denote a combination of the five instruments in period t as $\bar{\kappa}_t$, $\bar{\kappa}_t \equiv (\tau_t, \sigma_t, \eta_t, \theta_t, \xi_t)$.

Expressed as shares of GDP, public investment, net transfers to retirees, and net transfers to workers are given by

$$\begin{aligned} \text{investment share} &= (1-\alpha)(\sigma_t + \theta_t\alpha'), \\ \text{share of transfers to retirees} &= (1-\alpha)(\tau_t - (\eta_t + \theta_t)\alpha'), \\ \text{share of transfers to workers} &= (1-\alpha)(\eta_t\alpha' - (\tau_t + \sigma_t)), \end{aligned}$$

respectively.

3.3 Consumers

Consumers value young- and old-age consumption as well as leisure and discount the future at factor $\beta \in (0,1)$. To enable us to characterize the equilibrium in closed form, we assume that the period utility function of consumption is logarithmic. The indirect utility function of a young household in period t is then given by

$$\begin{aligned}
\max_{s_t, x_t} \quad & \ln(c_{1,t}) + v(x_t) + \beta \ln(c_{2,t+1}) \\
\text{s.t.} \quad & c_{1,t} = w_t(1-x_t)(1-\tau_t-\sigma_t-\xi_t) + a_t - s_t, \\
& c_{2,t+1} = s_t R_{t+1}(1-\eta_{t+1}-\theta_{t+1}) + b_{t+1}.
\end{aligned}$$

The felicity function of leisure is assumed to be strictly increasing and concave.

The first-order conditions characterizing the households' savings and labor-supply decisions are standard. Conditional on factor prices, tax rates, and benefits, the marginal rate of substitution between current and future consumption is equalized with the corresponding marginal rate of transformation, the after-tax gross interest rate. Similarly, the marginal rate of substitution between first-period consumption and leisure is equalized with the after-tax wage:

$$\begin{aligned}
\frac{1}{c_{1,t}} &= \beta R_{t+1}(1-\eta_{t+1}-\theta_{t+1})\frac{1}{c_{2,t+1}}, \\
v'(x_t) &= w_t(1-\tau_t-\sigma_t-\xi_t)\frac{1}{c_{1,t}}.
\end{aligned}$$

Due to our assumption of logarithmic preferences over consumption, the Euler equation characterizing the optimal savings choice of an *individual* household yields a closed-form solution for the *aggregate* savings function that maps the disposable income of a cohort as well as anticipated future tax rates into that cohort's savings:[4]

$$s_t = z(\tau_{t+1}, \eta_{t+1}, \theta_{t+1}) w_t(1-x_t)(1-\tau_t-\sigma_t+\eta_t\alpha'),$$

where we define

$$z(\tau_{t+1}, \eta_{t+1}, \theta_{t+1}) \equiv \frac{\alpha\beta(1-\eta_{t+1}-\theta_{t+1})}{\alpha(1+\beta)(1-\eta_{t+1}-\theta_{t+1})+(1-\alpha)\tau_{t+1}} \geq 0.$$

[4] To see this, note that the optimal savings choice of an individual consumer is characterized (from the Euler equation above) by

$$s_t R_{t+1}(1-\eta_{t+1}-\theta_{t+1}) + b_{t+1} = \beta R_{t+1}(1-\eta_{t+1}-\theta_{t+1})[w_t(1-x_t)(1-\tau_t-\sigma_t-\xi_t)+a_t-s_t].$$

Substituting for benefits and factor prices (and setting individual and aggregate savings equal to each other), we arrive at

$$\begin{aligned}
s_t R_{t+1}(1-\eta_{t+1}-\theta_{t+1}) + \nu w_{t+1}(1-x_{t+1})\tau_{t+1} &= \beta R_{t+1}(1-\eta_{t+1}-\theta_{t+1})[w_t(1-x_t)(1-\tau_t-\sigma_t+\eta_t\alpha')-s_t], \\
\Rightarrow (1-\eta_{t+1}-\theta_{t+1}) + \tau_{t+1}/\alpha' &= \beta(1-\eta_{t+1}-\theta_{t+1})[w_t(1-x_t)(1-\tau_t-\sigma_t+\eta_t\alpha')/s_t - 1].
\end{aligned}$$

The last equation yields a closed-form solution for the fixed-point problem, i.e., the aggregate savings function.

3.4 Economic Equilibrium

For convenience, we work with the state variables A_t and $q_t \equiv A_t^{1-\alpha} s_{t-1}^{\alpha}$ rather than the "original" state variables A_t and s_{t-1}. Substituting the expressions for wages and returns into the consumers' optimality conditions, the equilibrium allocation can recursively be expressed in terms of the following functions of policy instruments:

$$\left.\begin{array}{rcl} s_t & = & B_0(1-\alpha)\nu^{-\alpha}\ q_t\ (1-x_t)^{1-\alpha}\ (1-\tau_t-\sigma_t+\eta_t\alpha')z(\tau_{t+1},\eta_{t+1},\theta_{t+1}), \\ c_{1,t} & = & B_0(1-\alpha)\nu^{-\alpha}\ q_t\ (1-x_t)^{1-\alpha}\ (1-\tau_t-\sigma_t+\eta_t\alpha')(1-z(\tau_{t+1},\eta_{t+1},\theta_{t+1})), \\ c_{2,t} & = & B_0\nu^{1-\alpha}\ q_t\ (1-x_t)^{1-\alpha}\ (\alpha(1-\eta_t-\theta_t)+(1-\alpha)\tau_t), \\ x_t & = & x(\tau_t,\sigma_t,\eta_t,\xi_t,\tau_{t+1},\eta_{t+1},\theta_{t+1}), \\ A_{t+1} & = & B_1\ A_t^{1-\delta}\ q_t^{\delta}\ (1-x_t)^{\delta(1-\alpha)}\ \left(B_0\nu^{1-\alpha}((1-\alpha)\sigma_t+\alpha\theta_t)\right)^{\delta}, \\ q_{t+1} & = & B_0^{\delta(1-\alpha)+\alpha} B_1^{1-\alpha}(1-\alpha)^{\alpha}\nu^{\delta(1-\alpha)^2-\alpha^2}\ A_t^{(1-\delta)(1-\alpha)}\ q_t^{\delta(1-\alpha)+\alpha}\times \\ & & (1-x_t)^{\delta(1-\alpha)^2+\alpha(1-\alpha)}\times \\ & & (1-\tau_t-\sigma_t+\eta_t\alpha')^{\alpha} z(\tau_{t+1},\eta_{t+1},\theta_{t+1})^{\alpha}((1-\alpha)\sigma_t+\alpha\theta_t)^{\delta(1-\alpha)}. \end{array}\right\} \quad (2)$$

Here, the function $x(\cdot)$ is implicitly defined by the reduced first-order condition characterizing labor supply,

$$v'(x_t)(1-x_t)(1-z(\tau_{t+1},\eta_{t+1},\theta_{t+1})) = \frac{1-\tau_t-\sigma_t-\xi_t}{1-\tau_t-\sigma_t+\eta_t\alpha'}. \quad (3)$$

Note that labor supply in period t is independent of τ_t and σ_t if $\eta_t = \xi_t = 0$.

Conditional on initial values for the two state variables, (A_0, q_0), as well as a sequence of policy instruments, $\{\bar{\kappa}_t\}_{t=0}^{\infty}$, conditions (2) and (3) fully characterize the equilibrium allocation. Taking logarithms of the laws of motion of the two state variables, we can express these two equations as

$$\begin{bmatrix} \ln(A_{t+1}) \\ \ln(q_{t+1}) \end{bmatrix} = \underbrace{\begin{bmatrix} 1-\delta & \delta \\ (1-\alpha)(1-\delta) & \alpha+\delta(1-\alpha) \end{bmatrix}}_{M} \begin{bmatrix} \ln(A_t) \\ \ln(q_t) \end{bmatrix} + \underbrace{\begin{bmatrix} f^A(\cdot) \\ f^q(\cdot) \end{bmatrix}}_{f_t} \quad (4)$$

where the definitions of $f^A(1-x_t(\cdot),\sigma_t,\theta_t)$ and $f^q(1-x_t(\cdot),\tau_t,\sigma_t,\eta_t,\theta_t,\tau_{t+1},\eta_{t+1},\theta_{t+1})$ follow from (the logarithms of) the laws of motion in (2).

Inelastic Labor Supply We will sometimes consider the special case with inelastic labor supply, $v'(x) = 0$. In this special case, the equilibrium conditions (2) maintain their validity, but (3) is irrelevant and $x_t = 0$ for all t. Moreover, since the instrument ξ_t then has no effect on the allocation, we can normalize it to zero, $\xi_t = 0$ for all t.

3.5 Balanced Growth Path

On a balanced growth path, all policy instruments are constant over time, implying that per-capita labor supply is time-invariant as well. From (2), the growth rates of s_t, $c_{1,t}$, and $c_{2,t}$ then are equal to the growth rate of q_t. Moreover, the laws of motion for the two state variables in (2) imply that the gross growth rate of A_t, γ_A, must equal the gross growth rate of q_t on a balanced growth path. For any time-invariant choice of instruments, the last two equations in (2) therefore pin down the ratio A_t/q_t on the corresponding balanced growth path. Given this ratio, the same two conditions pin down γ_A and thus, the balanced growth rates of $q_t, s_t, c_{1,t}$, and $c_{2,t}$. Following these steps, we find

$$\gamma_A = \Big(B_0^\delta B_1^{1-\alpha}(1-\alpha)^{\alpha\delta}\nu^{\delta(1-2\alpha)}(1-x)^{\delta(1-\alpha)}(1-\tau-\sigma+\eta\alpha')^{\alpha\delta} \times z(\tau,\eta,\theta)^{\alpha\delta}((1-\alpha)\sigma+\alpha\theta)^{\delta(1-\alpha)}\Big)^{\frac{1}{1-\alpha(1-\delta)}} \text{ s.t. (3).}$$

3.6 Dependence Among Policy Instruments

Inspection of (2) and (3) reveals that the five policy instruments are not independent of each other:

Lemma 1. Consider a particular choice of contemporaneous policy instruments, $\bar{\kappa}_t = (\tau_t, \sigma_t, \eta_t, \theta_t, \xi_t)$, that satisfies (1). Fix the policy instruments implemented in the following period, $\bar{\kappa}_{t+1}$. Let $\mathcal{A}_t = (s_t, c_{1,t}, c_{2,t}, x_t, A_{t+1}, q_{t+1})$ be the contemporaneous equilibrium outcome implied by the initial condition (A_t, q_t), the policy instruments $\bar{\kappa}_t$ and $\bar{\kappa}_{t+1}$, as well as conditions (2), (3). (The latter condition only applies when labor supply is elastic.) Then, holding (A_t, q_t) and $\bar{\kappa}_{t+1}$ fixed, the same $\mathcal{A}_t$ is implied by a different choice of contemporaneous policy instruments, namely $\bar{\kappa}'_t = (\tau_t, \sigma_t - \eta_t\alpha', 0, \eta_t + \theta_t, \xi_t + \eta_t\alpha')$, where $\bar{\kappa}'_t$ also satisfies (1).

We can therefore normalize η_t to zero. Transfers from retirees to workers can fully be replicated by lower worker contributions to public investment in combination with higher retiree contributions to public investment and higher purely distortive labor taxes (this last component to ensure that the choice of leisure remains unaffected). Let κ_t denote the set of independent policy instruments in period t, $\kappa_t \equiv (\tau_t, \sigma_t, \theta_t, \xi_t)$, where we normalize η_t to zero from now on, for all t.

4 Ramsey Policy

Before analyzing the politico-economic equilibrium, we characterize the allocation resulting under the Ramsey policy. We assume that the Ramsey plan maximizes a weighted average of the welfare of all current and future cohorts. Welfare of future cohorts is discounted at the factor ρ. In all numerical examples, we assume $\rho = \beta\nu$, i.e., the Ramsey plan respects the time preference of households and accounts for population growth. The Ramsey policy is given as follows:

$$\max_{\{\kappa_s\}_{s=t}^{\infty}} G(A_t, q_t, \{\kappa_s\}_{s=t}^{\infty}) \text{ s.t. } (1),$$

where

$$\begin{aligned} G(A_t, q_t, \{\kappa_s\}_{s=t}^{\infty}) \quad &\equiv \quad \sum_{s=t}^{\infty} \rho^{s-t}(\beta \ln(c_{2,s}) + \rho \ln(c_{1,s}) + \rho v(x_s)) \\ \text{s.t.} \quad & \quad (2), (3) \text{ for all } s \geq t, \; A_t \text{ and } q_t \text{ given.} \end{aligned}$$

Denoting a typical term in the objective function by π_s, we have

$$\begin{aligned} \pi_s \quad &\equiv \quad \beta \ln(c_{2,s}) + \rho \ln(c_{1,s}) + \rho v(x_s) \text{ s.t. } (2), (3) \\ &= \quad \beta \ln[q_s(1 - x_s)^{1-\alpha}(\alpha(1 - \theta_s) + (1 - \alpha)\tau_s)] + \\ & \quad \rho \ln[q_s(1 - x_s)^{1-\alpha}(1 - \tau_s - \sigma_s)(1 - z(\tau_{s+1}, 0, \theta_{s+1}))] + \\ & \quad \rho v(x_s) + \text{constant terms} \text{ s.t. } (3) \\ &= \quad \ln(q_s)(\beta + \rho) + \ln(1 - x_s)(1 - \alpha)(\beta + \rho) + \rho v(x_s) + \\ & \quad \beta \ln(\alpha(1 - \theta_s) + (1 - \alpha)\tau_s) + \rho \ln(1 - \tau_s - \sigma_s) + \\ & \quad \rho \ln(1 - z(\tau_{s+1}, 0, \theta_{s+1})) + \text{constant terms} \text{ s.t. } (3). \end{aligned}$$

Consider the direct and indirect effects (the latter working through induced changes in q_s) on the objective function that are triggered by a marginal change in one of the policy instruments, ϕ_i say with $\phi_i \in \{\tau_i, \sigma_i, \theta_i, \xi_i\}$, $i \geq t$. The direct effect is given by

$$\begin{aligned} \frac{dG(A_t, q_t, \{\kappa_s\}_{s=t}^{\infty})}{d\phi_i}\Big|_{\text{dir}} \quad = \quad & \rho^{i-t}(\rho v'(x_i) - (1 - \alpha)(\beta + \rho)/(1 - x_i))\frac{\partial x_i}{\partial \phi_i} + \\ & \rho^{i-1-t}(\rho v'(x_{i-1}) - (1 - \alpha)(\beta + \rho)/(1 - x_{i-1}))\frac{\partial x_{i-1}}{\partial \phi_i} + \\ & \rho^{i-t}\frac{\partial \beta \ln[\alpha(1 - \theta_i) + (1 - \alpha)\tau_i] + \rho \ln(1 - \tau_i - \sigma_i)}{\partial \phi_i} + \\ & \rho^{i-1-t}\frac{\partial \rho \ln[1 - z(\tau_i, 0, \theta_i)]}{\partial \phi_i}, \end{aligned}$$

where the second and fourth lines only apply if $i > t$ as they capture effects of ϕ_i on choices in the preceding period, $i-1$.

The indirect effect is given by

$$\begin{aligned}
\frac{dG(A_t, q_t, \{\kappa_s\}_{s=t}^{\infty})}{d\phi_i}|_{\text{ind}} &= \sum_{s=t}^{\infty} \rho^{s-t}(\beta+\rho)\frac{\partial \ln(q_s)}{\partial \phi_i} \quad \text{s.t. (4)} \\
&= (\beta+\rho)\sum_{s=t}^{\infty} \rho^{s-t}\left[M^{s-1-i}\frac{\partial f_i}{\partial \phi_i} + M^{s-i}\frac{\partial f_{i-1}}{\partial \phi_i}\right]_{[2,\cdot]} \\
&= (\beta+\rho)\left[\rho^{i+1-t}\frac{\partial f_i}{\partial \phi_i}\sum_{k=0}^{\infty}(\rho M)^k + \rho^{i-t}\frac{\partial f_{i-1}}{\partial \phi_i}\sum_{k=0}^{\infty}(\rho M)^k\right]_{[2,\cdot]} \\
&= (\beta+\rho)\rho^{i-t}\left[(I-\rho M)^{-1}\right]_{[2,\cdot]}\left(\rho\frac{\partial f_i}{\partial \phi_i} + \frac{\partial f_{i-1}}{\partial \phi_i}\right),
\end{aligned}$$

where, by convention, matrices with a negative exponent equal zero. For the same reason as above, the term $\partial f_{i-1}/\partial \phi_i$ only applies if $i > t$.

4.1 Inelastic Labor Supply

With inelastic labor supply, the expressions for the direct and indirect effects on $G(\cdot)$ simplify as x_i is unaffected by policy changes. Several constellations may arise:

i. Interior optimum. An interior equilibrium in period t exists if κ_t solves any two out of the three relevant[5] first-order conditions in period t with equality, and if it satisfies (1). Solving the first-order conditions in period t, we find that τ_t and σ_t are functions of θ_t. Evaluated at $\theta_t = 0$, the tax rates are given by

$$\begin{aligned}
\tau &= \frac{\beta(1-\rho-\alpha(1-(1-\delta)\rho^2)) - \alpha\rho(1-(1-\delta)\rho)}{(1-\alpha)(\beta+\rho)(1-(1-\delta)\rho)}, \\
\sigma &= \frac{\delta\rho}{1-(1-\delta)\rho}.
\end{aligned}$$

At those tax rates (but not the tax rates resulting when evaluated at $\theta_t \neq 0$), all three first-order conditions in period $i > t$ are also satisfied with equality,[6] implying that the

[5]For any κ_t,

$$\alpha'\left(\frac{\partial G(\cdot)}{\partial \sigma_t} - \frac{\partial G(\cdot)}{\partial \tau_t}\right) \equiv \frac{\partial G(\cdot)}{\partial \theta_t}.$$

This condition need not hold for $i > t$ though.

[6]At the tax rates evaluated at $\theta = 0$,

$$\alpha'\left(\frac{\partial G(\cdot)}{\partial \sigma_i} - \frac{\partial G(\cdot)}{\partial \tau_i}\right) \equiv \frac{\partial G(\cdot)}{\partial \theta_i} \text{ for any } \kappa_i,\ i > t.$$

time-invariant tax rates $\theta = 0$ and τ and σ given above constitute the interior solution to the program. Depending on parameter values, these tax rates need not satisfy (1).

ii. Corner solution for τ. A corner solution for τ_i (but not σ_i and θ_i) arises if the constraint $\tau_i \geq 0$ is binding, while the constraints $\sigma_i + \theta_i \alpha' \geq 0$ as well as $1 \geq \tau_i + \sigma_i \geq 0$ and $1 \geq \theta_i \geq 0$ do not bind. (Due to the dependence of the marginal conditions in period t (see footnote 5) this constellation cannot arise in period t; while $\tau_t = 0$, the constraint $\tau_t \geq 0$ does not bind.) This situation is characterized by $\tau_i = 0$, together with the first-order conditions with respect to σ_i and θ_i holding with equality.

Solving the first-order conditions with respect to σ_i and θ_i in period i and fixing τ_i at zero, we find

$$\begin{aligned} \sigma &= \frac{\beta(\rho\alpha + \alpha - 1)((\delta - 1)\rho + 1) + \rho(2\delta\rho\alpha - \rho\alpha + \alpha - \delta\rho)}{(\alpha - 1)(\beta + \rho)((\delta - 1)\rho + 1)}, \\ \theta &= \frac{\alpha\rho((\delta - 1)\rho + 1) + \beta\left(\rho + \alpha\left((\delta - 1)\rho^2 + 1\right) - 1\right)}{\alpha(\beta + \rho)((\delta - 1)\rho + 1)}. \end{aligned}$$

The time-invariant tax rates $\tau = 0$ and σ and θ given above therefore constitute a solution if $\partial G(\cdot)/\partial \tau_i < 0$ at these tax rates. Depending on parameter values, this may or may not be the case.

There exists a "critical" ρ, $\rho^c = -\frac{\alpha+\beta-\sqrt{(\alpha+\beta)^2-4(\alpha-1)\alpha\beta(\beta+1)(\delta-1)}}{2\alpha(\beta+1)(\delta-1)}$, such that for $\rho \leq \rho^c$ the interior regime applies, while for $\rho > \rho^c$ the corner solution regime applies. c_2/c_1 falls in constrained region where $\theta > 0$.

5 Politico-Economic Equilibrium

Retirees and workers vote on candidates representing platforms with values for the four policy instruments κ_t (no commitment, multidimensional policy space). Objective function derived from probabilistic voting setup. Per-capita weights of retirees and workers are given by ω^2 and ω^1, respectively. Let $\omega \equiv \omega^2/\omega^1$. When determining the policy instruments to implement in the current period, the political process anticipates the effects on current and future equilibrium outcomes, both economic and political. In a Markovian equilibrium, future leisure choice and policy choices are functions of the state variables, $x_{t+1} = \tilde{x}(A_{t+1}, q_{t+1})$ and $\kappa_{t+1} = \kappa(A_{t+1}, q_{t+1})$.

Conditional on anticipated policy- and leisure-choice functions $\kappa(\cdot)$ and $\tilde{x}(\cdot)$, the program solved by the political decision makers is

$$\max_{\kappa_t} W(A_t, q_t, \kappa_t; \kappa(\cdot), \tilde{x}(\cdot)) \quad \text{s.t. } (1),$$

where

$$\begin{aligned} W(A_t, q_t, \kappa_t; \kappa(\cdot), \tilde{x}(\cdot)) &\equiv \omega \ln(c_{2,t}) + \nu[\ln(c_{1,t}) + v(x_t) + \beta \ln(c_{2,t+1})] \\ \text{s.t.} \quad & (2), (3), \;\; A_t \text{ and } q_t \text{ given,} \\ & \kappa_{t+1} = \kappa(A_{t+1}, q_{t+1}), \;\; x_{t+1} = \tilde{x}(A_{t+1}, q_{t+1}). \end{aligned}$$

Economic equilibrium dictates that the anticipated leisure choice function $\tilde{x}(\cdot)$ is consistent with the optimality condition (3), for any combination of state variables, i.e.,

$$\tilde{x}(A, q) \equiv x(\kappa(A, q), \tau(A', q'), \theta(A', q'))$$

subject to the equilibrium law of motion of the state variables. Furthermore, political equilibrium dictates that for any combination of state variables (A_t, q_t), the κ_t solving the above program is given by $\kappa(A_t, q_t)$.

Using the equilibrium expressions for consumption from (2), and omitting terms unaffected by current and future policy choices, the political objective can be written as

$$\begin{aligned} W(\cdot) = \; & \omega \ln[(1 - x_t)^{1-\alpha}(\alpha(1 - \theta_t) + (1 - \alpha)\tau_t)] + \\ & \nu\{\ln[(1 - x_t)^{1-\alpha}(1 - \tau_t - \sigma_t)(1 - z(\tau_{t+1}, 0, \theta_{t+1}))] + v(x_t) + \\ & \beta \ln[q_{t+1}(1 - \tilde{x}(A_{t+1}, q_{t+1}))^{1-\alpha}(\alpha(1 - \theta_{t+1}) + (1 - \alpha)\tau_{t+1})]\}, \end{aligned}$$

where A_{t+1} and q_{t+1} follow from (2) and x_t solves (3).

In light of the fact that we found the tax rates under the Ramsey policy to be independent of the state variables, we guess that the same holds true for the political equilibrium choices. (XXX Uniqueness...) We verify this guess if we find that constancy of $\kappa(\cdot)$ and thus, $\tilde{x}(\cdot)$ imply that the κ_t solving the above program as well as the equilibrium labor supply are indeed independent of the current state variables.

Imposing the guess and omitting terms unaffected by current and future policy choices if the guess is correct, the objective function reduces to

$$\begin{aligned} W(\cdot) = \; & \omega \ln[(1 - x_t)^{1-\alpha}(\alpha(1 - \theta_t) + (1 - \alpha)\tau_t)] + \\ & \nu\{\ln[(1 - x_t)^{1-\alpha}(1 - \tau_t - \sigma_t)] + v(x_t) + \\ & \beta \ln[(1 - x_t)^{\delta(1-\alpha)^2 + \alpha(1-\alpha)}(1 - \tau_t - \sigma_t)^{\alpha}((1 - \alpha)\sigma_t + \alpha\theta_t)^{\delta(1-\alpha)}]\} \text{ s.t. } (3). \end{aligned}$$

Omitting more irrelevant terms, the political objective function can now be written as

$$\begin{aligned} W(\cdot) &= \omega \ln[\alpha(1-\theta_t) + (1-\alpha)\tau_t] + \\ & \quad \nu\{\ln(1-\tau_t-\sigma_t) + \beta \ln[(1-\tau_t-\sigma_t)^{\alpha}((1-\alpha)\sigma_t + \alpha\theta_t)^{\delta(1-\alpha)}]\}. \end{aligned}$$

The same reduced objective function results (under the guess) in the case with inelastic labor supply.

5.1 Inelastic Labor Supply

Parallel to Ramsey case, the effects of marginal policy changes are linearly dependent.[7] Several constellations may arise:

i. Interior equilibrium.

$$\begin{aligned} \tau &= \frac{\beta\nu(-\theta\delta+\delta+\theta-1)\alpha^2 + (\beta\delta\nu+\nu+\omega)(\theta-1)\alpha+\omega}{(\alpha-1)((\alpha\beta(\delta-1)-\beta\delta-1)\nu-\omega)}, \\ \sigma &= \frac{\beta\nu\left((\alpha-1)\delta(\alpha\theta-1)-\alpha^2\theta\right)-\alpha(\nu+\omega)\theta}{(\alpha-1)((\alpha\beta(\delta-1)-\beta\delta-1)\nu-\omega)}. \end{aligned}$$

θ is *not* pinned down. Shares, but not growth rate independent of θ. Growth rate maximized at $\theta = 0$.

ii. Corner solution.

$$\begin{aligned} \sigma &= \frac{\beta(\delta-1)\nu\alpha^2 - (2\beta\delta\nu+\nu+\omega)\alpha+\beta\delta\nu+\omega}{(\alpha-1)((\alpha\beta(\delta-1)-\beta\delta-1)\nu-\omega)}, \\ \theta &= \frac{\beta(\delta-1)\nu\alpha^2-(\beta\delta\nu+\nu+\omega)\alpha+\omega}{\alpha((\alpha\beta(\delta-1)-\beta\delta-1)\nu-\omega)}. \end{aligned}$$

There exists a "critical" ν, $\nu^c(\theta) = \frac{\omega(\alpha(\theta-1)+1)}{\alpha(\alpha\beta(\delta-1)-\beta\delta-1)(\theta-1)}$, such that for $\nu \le \nu^c$ the interior regime applies, while for $\nu > \nu^c$ the corner solution regime applies.

5.2 Elastic Labor Supply

5.3 Calibration

Our model allows for a direct estimation of the growth gap that can be attributed to the "political failure" of decision making under majority voting. To this effect we calibrate our model to replicate the postwar growth performance of US economy. We set $\alpha = 0.2815$; $\beta = 0.48846$;

[7] The condition given in footnote 5, with $G(\cdot)$ replaced by $W(\cdot)$, applies.

$\nu = 1.3843$ the gross growth rate of the U.S. population between 1970 and 2000 (1.384).[8] We also set $\delta(1-\alpha) = 0.2$, according to estimates of long-run elasticity of output to infrastructure investment. This gives a political choice of $\tau = 0.121$ and $\sigma = 0.069$. The Ramsey plan calls for a much larger public investment, financed both by the young and old. Taxes are $\theta = 0.187$ and $\sigma = 0.367$. If we calibrate B_0 and B_1 such that γ_A for the politico-economic outcome replicates the actual US postwar productivity growth (which in annual terms is 1.26%), we find that the growth cost with respect to the Ramsey plan is a huge 1.55% per year.

[8]Piketty and Saez find α to vary between 0.68 and 0.75 in post-war U.S. data. The population growth rate is reported by the U.S. Census Bureau.

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