Sticky Prices and the Optimal Return to Money

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Motivation

• Traditional view on Business Cycles and Money: Money matters!
  
  – need devices to break Classical Dichotomy: signal extraction problem, menu costs, nominal contracts, segmented markets.

  – Lucas (1972): monetary policy is noisy.


• Our view: correlations between monetary and real variables are not accidental but the result of frictions in the real sector that money alleviates.
**What we do:**

- Introduce aggregate uncertainty into a standard search model of money.

- Study optimal allocations (mechanism design problem).

- Show that the return to money (price level) is history dependent in optimal allocations.

**Literature:** Spear and Srivastava (1987) and Green (1987)... but the recursive structure for discussing non-stationary allocations in monetary models with heterogeneous agents has not been established. We therefore start simple!
Environment: Shi-Trejos-Wright with aggregate uncertainty.

1. Discrete time, discount factor $\beta$.

2. Specialization in production and consumption: $N$ types.

3. Money is indivisible $m \in \{0, 1\}$.

4. Divisible production $y$.

5. Taste-shocks $u_s(y)$, where $s \in \{low, high\}$ with probability $\pi_s$. 
Definitions.

- A history is \( s^t = (s^{t-1}, s_t) \). Set of all possible histories up to \( t \) is \( S^t \).

- Allocation is sequence \( y_t : S^t \rightarrow R \) or \( y(s^t) \) exchanged for money.

- Welfare Criteria

\[
\sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t p(s^t) z_{s_t}(y(s^t))
\]

where

\[
z_s(y) \equiv m(1 - m) \frac{1}{N} (u_s(y) - y).
\]

- First best allocation \((y^*_{l}, y^*_{h})\) such that \( u'_{s_t}(y(s^t)) = 1 \).
Expectations:

\[
v_1(s^t) = (1-m)\frac{1}{N}[u_{s^t}(y(s^t)) + \beta(\pi_l v_0(s^t, l) + \pi_h v_0(s^t, h))] + \\
(1-(1-m)\frac{1}{N})\beta(\pi_l v_1(s^t, l) + \pi_h v_1(s^t, h))
\]

\[
v_0(s^t) = m\frac{1}{N}[-y(s^t) + \beta(\pi_l v_1(s^t, l) + \pi_h v_1(s^t, h))] + \\
(1-m\frac{1}{N})\beta(\pi_l v_0(s^t, l) + \pi_h v_0(s^t, h)).
\]

Denote:

\[
\partial v(s^t) \equiv v_1(s^t) - v_0(s^t).
\]
Implementability and Optimality

The producer’s participation constraint is

\[ y(s^t) \leq \beta (\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)) \]

The consumer’s participation constraint is

\[ u_s(y(s^t)) \geq \beta (\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)). \]

Definitions:

1. An output allocation \( y(s^t) \) is implementable if there exists \( v(s^t) \) satisfying participation constraints for all \( s^t \in S^t \) and all \( t = 0, 1, 2, \ldots \).

2. An allocation is optimal if it maximizes welfare among the set of implementable allocations.
Promise keeping (rational expectations)

Return on money links $\partial v(s^t)$ to $\partial v(s^t, s_{t+1})$ as follows

$$\partial v(s^t) = f_{s^t}(y(s^t)) + (1 - \frac{1}{N})\beta(\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)),$$

where

$$f_s(y) \equiv \frac{1}{N}((1 - m)u_s(y) + my).$$
The sequential Planner’s problem

\[
\begin{align*}
\text{max} & \quad \sum_{t=0}^{\infty} \sum_{s_t \in S^t} y(s^t) \cdot \beta^t z_{s_t}(y(s^t)) \\
\text{s.t.} & \quad y(s^t) \leq \beta (\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)) \\
& \quad \partial v(s^t) \leq f_{s_t}(y(s^t)) + \alpha (\pi_l \partial v(s^t, l) + \pi_h \partial v(s^t, h)) \\
& \quad 0 \leq \partial v(s^t) \leq B \text{ for all } s^t \text{ and all } t.
\end{align*}
\]

**Note:** we ignore consumer’s participation constraint


No Ponzi Games

The constraint $\partial v(s^t) \leq B$ implies that the return to money is bounded above by the discounted-expected utility gain of having one unit of money

$$\partial v(s^t) \leq f_{s^t}(y(s^t)) + \sum_{\tau > t} \sum_{s^\tau \in S^\tau} \alpha^{\tau-t} p(s^\tau) f_{s^\tau}(y(s^\tau)).$$

Proposition 1 (Maximum Sustainable Debt) Any sequence $\{y(s^t), \partial v(s^t)\}$ satisfying the constraints of the Planner’s problem, is such that $\partial v(s^t) \leq \bar{d}_s$ for all $s^t$, where $\bar{d}_s$ solves $\bar{d}_s = f_s(\beta \bar{d}) + \alpha \bar{d}$ and $\bar{d} = \pi_l \bar{d}_l + \pi_h \bar{d}_h$ for $s \in \{l, h\}$. 
If we associate the multipliers $\beta^t p(s^t) \mu(s^t)$ and $\beta^t p(s^t) \lambda(s^t)$ to the producer’s and debt constraints, the FOC with respect to $\partial v(s^t, s')$ yields

$$\lambda(s^t, s_{t+1}) = \mu(s^t) + \left(1 - \frac{1}{N}\right)\lambda(s^t).$$

- Note that $\lambda(s^t, l) = \lambda(s^t, h)$
- Debt is unrestricted in the initial period: $\lambda(s_0) = 0$.
- History dependence requires $\mu(s^t) > 0$.
- When $\mu(s^t) = 0$, we have $\lambda(s^t, s_{t+1}) = \left(1 - \frac{1}{N}\right)\lambda(s^t) < \lambda(s^t)$. The rate of decay depends on $\frac{1}{N}$ (matching friction).
The state is \((s, d_l, d_h)\) .... but return \(d_s\) is the only relevant promise in realization \(s\). We thus write \((s, d)\), where \(d\) is a short for \(d_s\).

**Bellman’s equation**

\[
T_w(s, d) = \max_{y, d'_l, d'_h} \ z_s(y) + \beta (\pi_l w(l, d'_l) + \pi_h w(h, d'_h)) \\
\text{s.t.} \\
y \leq \beta (\pi_l d'_l + \pi_h d'_h) \\
d \leq f_s(y) + \alpha (\pi_l d'_l + \pi_h d'_h)
\]

**Proposition 2** Let \(w \in W\). Then, \(T_w\) is continuous, weakly decreasing in \(d\), and concave. The Bellman’s equation has a unique solution. Principle of Optimality applies.
Economy with no aggregate-uncertainty

Proposition 3 (No memory)  *In the economy without shocks, the optimal allocation is constant (no dynamics) and the consumer constraint slacks. First best allocation $y^*$ is only attained when $\beta$ is close to 1.*

Lesson 1: aggregate uncertainty is necessary for history-dependence.

Proposition 4 (Artificial dynamics.) *Fixed an initial $d_0$.\n
i) If $\beta$ is low, so that $y^* > \bar{y}$, no dynamics: $y(s^t) = \bar{y}$ and $d(s^t) = \bar{d}$.

ii) If $\beta$ is high, so that $y^* < \bar{y}$, debt and output converge monotonically to $d^*$ and $y^*$ for all initial $d_0 \in (d^*, \bar{d})$.

Lesson 2: history-dependence requires that producer’s constraint bind ... but not always (so that we can borrow from future states)
Economy with aggregate-uncertainty

Main result: for producer constraint to bind, but not always, discount factor should be not too high and not too low.

Proposition 5  Assume $\beta$ high enough so that $y_h^* < \beta d^*$, where
\[d^* = \frac{1}{1-\alpha}[\pi_h f_h(y_h^*) + \pi_l f_l(y_l^*)].\]
Then, the optimum is given by First-Best allocation $y_s^*$ and is thus not history-dependent.

Proposition 6  There exists $\beta$ such that the following holds. The values of $d^*$ and $\bar{d}$ satisfy $y_l^* < \beta d^* < y_h^* < \beta \bar{d}$ and, moreover, the optimum is history-dependent.
More on the economy with aggregate-uncertainty

**Proposition 7** There exists $\beta_0$ so that, when $\beta \leq \beta_0$, for which output is constant $y(s, d) = \hat{y} \leq y_i^*$ for all $(s, d)$. Moreover, output equals $y_i^*$ only if $\beta = \beta_0$.

Key insight: Since participation constraints bind in all states, the Planner cannot exploit inter-temporal trade-offs to induce more production when $s$ is high.
• LW economy with aggregate-taste shocks at beginning of each period.

• Day: decentralized market with anonymous bilateral matching.

• Night: centralized market where a general good is produced and exchanged.

• Preferences: \( u_s(y) - h + U(Y) - H. \)

• Growth rate of money \( \tau(s^t). \)
Mechanism design

Trading mechanisms have 2 components:

1. actions sets (include autarkic allocation).

2. outcome functions.

The mechanism we consider has 2 parts:

1. Day-trading mechanism: divide the pie.

Assume lump sum taxes are available

**Result 1.** For all $\beta > 0$ the first best level of output is implementable with counter-cyclical money-growth rates: $\tau_h < \tau_l$ and $\tau_h < 0$.

**Lump sum taxes are not available**

**Result 2.** The first best level of output is implementable if $\beta$ is close to 1. Moreover, optimality requires positive inflation in low state.

**Main Lesson:** price stickiness results from the absence of markets that give fiscal and monetary policy the ability to implement the first best.
Monitoring: non-monetary mechanisms.

- Any individual deviation can be detected and defectors punished with autarky.

- Full monitoring: whole history of individuals can be recorded.

- Limited monitoring: Planner can only record whether an individual has defected or not in the past

**Main Lesson**: Efficient allocations with limited monitoring are not history-dependent. With full monitoring history dependence can help relax incentive constraints.
Conclusions: Memory and 2nd Best Efficiency.

• We do no need "special assumptions" such as signal extraction problem, segmented markets, or nominal "rigidities".

\[
\begin{align*}
&\text{anonymity} \\
&\text{lack of commitment} \\
&\text{aggregate uncertainty}
\end{align*}
\] \Rightarrow \text{memory is a “natural” property of money.}

• Theory \Rightarrow \text{Money and Business Cycles are intertwined (propagation of shocks).}
The optimal allocation is described with the help of threshold debt levels \((\hat{d}_l, \hat{d}_h)\) such that:

1. In state \(s = l\), \(y(l, d) = y^*_l\) and \((d'_l(l, d), d'_h(l, d)) = (\hat{d}_l, \hat{d}_h)\) for all \(d \leq \hat{d}_l\). Output and new debt are increasing functions of \(d_l\). Moreover, the policy function for new debt when \(s = l\) is such that, for \(d_0\) on a right neighborhood of \(\hat{d}_l\), the sequence \(d^{n+1} = (d'_l(l, d^n), d'_h(l, d^n))\) is a decreasing sequence converging to \((\hat{d}_l, \hat{d}_h)\).

2. In state \(s = h\), for \(d_h \leq \hat{d}_h\), output is \(y^*_l < y_h < y^*_h\) and new debt is \((d'_l(h, d), d'_h(h, d)) > (\hat{d}_l, \hat{d}_h)\). Moreover, output and new debt are increasing functions of \(d_h\) for \(d_h\) in a right neighborhood of \(\hat{d}_h\).