Why Tax Capital?

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Abstract

We study optimal capital taxation in a limited commitment environment. Our environment consists of a continuum of households with idiosyncratic labor shocks, who have access to a complete contingent claims market. Financial contracts are not perfectly enforceable; as in Kehoe and Levine (1993), enforcement constraints take the form of endogenous debt limits. This market imperfection drives the endogenous discrepancy between the household and planner discount factors: households face the possibility of being debt constrained in the future, and as a result have a higher discount factor than the planner, who does not face such a constraint. In such an economy, the planner will choose an optimal capital level that is lower than that chosen by households; this difference in the choice of capital motivates imposing a positive capital income tax on households to induce them to invest at the socially optimal amount.

1 Introduction

In the Ramsey literature of capital taxation, Chamley (1986) and Judd (1985) argue for zero capital taxation in the long run. Chari and Kehoe (1999) show that the capital tax

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rate should be high initially and decrease to zero. Moreover, Atkeson, Chari and Kehoe (1999) show that the zero capital taxation result is robust for a wide range of assumptions. Finally, Lucas (1990) argues that for the U.S. economy, there is a significant welfare gain to be realized in switching to this policy. In sum, the zero capital taxation argument suggests that the current capital stock in the U.S. economy is too low because the capital tax rate is too high, and that decreasing the tax rate would lead to large welfare gains.

A competing literature argues that certain frictions can rationalize capital taxation. Aiyagari (1996) shows that with incomplete markets, agents have a precautionary savings motive that leads them to overinvest in capital. He proves that the optimal capital income tax should be positive in the long run to reduce over-investment. Golosov, Kocherlakota and Tsyvinski (2003) obtain a nonzero optimal capital taxation result by introducing incentive constraints, which arise from private information about an individual's idiosyncratic shock. To motivate high-skilled agents to reveal their type, they argue that the tax burden on these agents should be lighter than on low-skilled agents.

We study a different type of economy that is closely related to the recent literature[1] of endogenous incomplete markets, in which there is a continuum of households with idiosyncratic shocks and a complete set of contingent claims, but financial contracts are not perfectly enforceable. We construct the enforcement constraints to capture some features of the current US bankruptcy code: if households default on financial contracts, they have their assets seized by creditors; however, they continue to consume their labor income.. This paper solves the Ramsey planner's problem using the dual approach so that the Ramsey planner chooses sequences of after-tax wage rates, government consumption, aggregate capital and market interest rates that are consistent with a competitive equilibrium in order to maximize social welfare of the economy.

As in Kehoe and Levine (1993) and Alvarez and Jermann (2000), enforcement constraints take the form of endogenous debt limits. The imperfect enforceability of financial contracts drives the endogenous discrepancy between the household and planner discount factors[2]: households face the possibility of being debt constrained in the future, and as a result have a higher discount factor than the planner, who does not face such a constraint (as in Aiyagari (1995)). In such an economy, the planner will choose an optimal capital level that is lower

[1] Alvarez and Jermann (2000,2001), Kehoe and Levine (1993), Kehoe and Perri (2002,2004), Kocherlakota (1996), Krueger (1999), Lustig (2005) among others.

[2] Alvarez and Jermann (2000), Kehoe and Levine (1993) and Lustig (2005) show endogenous borrowing limits lower the market interest rate.

than that chosen by households. As in Aiyagari (1995), this difference in the choice of capital motivates imposing a positive capital income tax on households to induce them to invest at the socially optimal amount. However, it is not the missing market, but the market imperfection–financial friction– that drives our results.

We show that a positive rate of capital taxes is optimal in an economy where the market interest rate is lower than the planner's. As we will show later, the dual approach of Ramsey problem enables the government to choose sequences of aggregate capital and government consumption without affecting the sequence of competitive equilibrium prices. In other words, the optimal capital level should satisfy the modified golden rule without creating any externalities; this implies that the government's steady state discount factor becomes the time discount factor regardless of the market settings. Hence, we argue that our positive capital taxation should hold with any model economy that delivers a lower market interest rate than the government's.

This paper also addresses the role of labor income tax and to find an optimal labor tax. In a debt-constrained environment, the labor tax helps relax the household's debt limits. On the other hand, the labor income tax also creates a distortion. The Ramsey planner takes both effects into consideration and chooses an optimal labor tax such that in the aggregate, the marginal benefit of relaxing the debt limit is equal to the marginal cost of distortions due to lost resources.

This paper is organized as follows. Section 2 describes our model economy and characterizes the competitive equilibrium. Section 3 describes the enforcement technology in the economy. Section 4 defines the competitive equilibrium. Section 5 solves and characterizes the Ramsey problem. Section 6 analyzes the steady state. Section 7 discusses optimal capital tax in the long run. Section 8 carries out the quantitative analysis for our calibrated model. Section 9 concludes.

2 Environment

There is a continuum of agents of measure one. Agents start with an initial asset holding, a_0 and an initial idiosyncratic labor shock, s_0.The initial joint distribution over (a_0, s_0) is given by Φ_0. There is a single, non-storable, consumption good. A household's preferences are described by the expected value of the sum of discounted utilities of private consumption and public consumption:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\middle|s_0\right)\left[u\left(c_t\right)+U\left(G_t\right)\right]$$

where c_t, G_t denote private consumption and public consumption respectively. $\beta \in (0,1)$, $u(\cdot)$ is the utility from private consumption, and $U(\cdot)$ is the utility from public consumption. The functions $u(\cdot)$ and $U(\cdot)$ are each assumed to be bounded, continuously differentiable, strictly increasing, and strictly concave.

There is no aggregate uncertainty. The only event that each household faces is a stochastic idiosyncratic market labor productivity shock s. Each event s_i takes values on a discrete grid $S \equiv \{s_1, ..., s_i, ..., s_I\}$. The idiosyncratic shock $\{s\}$ follows a Markov process, with a transition probability $\pi(s'|s)$. We assume that the law of large numbers holds, so that transition probabilities can be interpreted as fractions of agents making the transition from one state to another. In addition, we assume that there is a unique invariant distribution $\bar{\Pi}(s)$. By the law of large numbers, $\bar{\Pi}(s)$ is the fraction of agents in state s in each period. We denote s^t as a history of realization of the shock:

$$s^t = (s_0, s_1, ..., s_{t-1}, s_t)$$

Households are endowed with one unit of perfectly divisible labor for each period, which can be used either in the market sector or home sector. Let l_t and $1 - l_t$ be the amount of labor the household allocates to market and to home production at period t, respectively. Home production is given by the following production function:

$$H(1 - l_t),$$

where $H : [0,1] \to \mathbb{R}_+$ is bounded, continuously differentiable, strictly increasing, and strictly concave. In addition, $H(\cdot)$ satisfies $H(0) = 0, H'(0) = \infty$, and $H'(1) > 0$.

In the market sector, market production is given by a single technology that exhibits constant returns to scale:

$$Y_t = F(K_t, L_t)$$

where $F(\cdot,\cdot)$ is the market production function, and K_t and L_t denote aggregate capital and aggregate labor inputs respectively. Assume that $F(\cdot,\cdot)$ is homogeneous of degree one, and twice continuously differentiable.

There are competitive markets in labor, capital, the output good, and government bonds.

3 Enforcement Technology

Following Kehoe and Levine (1993) and Kocherlakota (1996), this literature commonly assumes that when households default, they are excluded from financial markets forever, and have their assets seized at the time of default. This punishment forces households to consume only their total labor income-the sum of home production and market wage income-forever, which implies, in turn, that the per-period utility that a household obtains after default is

$$U_{aut}\left(s_t, \overline{w}_t\right) \equiv u(H\left(1-l_t\right) + \overline{w}_t s_t l_t)$$

where $\overline{w}_t$ and l_t are after tax wage rate. We can thus define an autarky value at period t as

$$V_{aut,t}(s_t, \{\overline{w}_\tau\}_{\tau=t}^{\infty}) \equiv \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \succeq s^t} \pi\left(s^\tau \middle| s^t\right) U_{aut}\left(s_t, \overline{w}_t\right).$$

This is the autarky value that a household receives if it defaults at period t.

To keep households from defaulting, we need to ensure that the expected utility of staying in the risk-sharing pool are greater than or equal to the value of defaulting for each possible s^t. The enforcement constraints can be written as follows:

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau \succeq s^t} \beta^{\tau-t} \pi\left(s^\tau \middle| s^t\right) u\left(c_\tau\right) \geq \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \succeq s^t} \pi\left(s^\tau \middle| s^t\right) U_{aut}\left(s_t, \overline{w}_t\right) \text{ for } \forall s^t$$

In other words, if this constraints are satisfied in all states, households do not wish to exercise their default option.

4 Competitive Equilibrium

In this section we formulate the household and firm problems, and define an equilibrium, in which all trading occurs at period zero.

4.1 Household (a_0, s_0)'s Problem

Taking the sequence of after-tax wages $\overline{w}_t$ and market interest rates R_t as given, a household purchases history-contingent consumption claims $\{c_t\}$ and makes labor allocation decisions $\{l_t\}$ subject to both a lifetime budget constraint and a sequence of enforcement constraints, one for each history:

$$W_0\left(a_0, s_0, \{\overline{w}_t\}_{t=0}^{\infty}, \{R_t\}_{t=0}^{\infty}\right) \equiv \max_{\{c_t, l_t\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left(c_t\right),$$

subject to

$$\sum_{t=0}^{\infty} \sum_{s^t} \prod_{s=1}^{t} \frac{1}{R_{s-1}} \pi\left(s^t \middle| s_0\right) \left[H\left(1-l_t\right) + \overline{w}_t s_t l_t - c_t\right] \geq -a_0 \tag{1}$$

$$\sum_{\tau=t}^{\infty} \sum_{s^\tau \succeq s^t} \beta^{\tau-t} \pi\left(s^\tau \middle| s^t\right) u\left(c_\tau\right) - \sum_{\tau=t}^{\infty} \sum_{s^\tau \succeq s^t} \beta^{\tau-t} \pi\left(s^\tau \middle| s^t\right) U_{aut}\left(\overline{w}_\tau, s_\tau\right) \geq 0 \text{ for } \forall\ s^t,\ t \geq 0 \tag{2}$$

As mentioned above, a household's labor income consists of two parts: home production and market wage income. The solution to the labor allocation problem is obtained by following a static optimization problem:

$$\max_{l_t \in [0,1]} H\left(1-l_t\right) + \overline{w}_t s_t l_t$$

This total labor income maximization problem yields an optimal supply function for market work, denoted $l\left(s_t, \overline{w}_t\right)$. Using this, we can define a household's total maximized labor income function (denoted by $y\left(s_t, \overline{w}_t\right)$) as

$$y\left(s_t, \overline{w}_t\right) \equiv H\left(1 - l\left(s_t, \overline{w}_t\right)\right) + \overline{w}_t s_t l\left(s_t, \overline{w}_t\right)$$

4.2 Firms' Problem

Firms operate market production technology through a market production function, $F(K_t, L_t)$. At period 0, taking a sequence of pre-tax wage rates $\{w_t\}$, market interest rates $\{R_t\}$, and corporate profit taxes τ_K as given, a firm finances its initial capital stock K_0 using financial intermediaries and chooses a sequence of capital stocks K_{t+1} and labor demand L_t that maximizes the discounted after-tax profit function:

$$\max_{\{K_{t+1}, L_t, I_t, \varphi_t\}} \sum_t \prod_{s=1}^{t} \frac{1}{R_{s-1}} \left[\left(1-\tau_{K,t}\right)\varphi_t - I_t + \delta \tau_{K,t} K_t\right]$$

subject to

$$\begin{aligned} \varphi_t &= F\left(K_t, L_t\right) - w_t L_t \\ K_{t+1} &= \left(1-\delta\right) K_t + I_t \end{aligned}$$

where φ_t is corporate profit (capital income) and $\overline{w}_t$ equals $(1-\tau_{L,t})w_t$.

Note that it is firms that must pay the tax, which is imposed on income paid to physical capital.

4.3 Financial Intermediaries and Government

Financial intermediaries trade a sequence of contingent claims and government bonds, and finance initial capital stock K_0 for firms.

Government expenditure is composed of government consumption G_t and debt payments $(r_t^b+1)B_t$. Government revenue is composed of taxes on market labor income and capital returns, respectively labeled $\tau_{K,t}$ and $\tau_{L,t}$. Additionally, the government can finance its expenditures by issuing new debt B_{t+1}. Hence, the government constraint is as follows:

$$\tau_{k,t}(r_t-\delta)K_t+\tau_{l,t}w_tL_t+B_{t+1}=(r_t^b+1)B_t+G_t$$

4.4 Characterizing Equilibrium Prices and Allocations

Household (a_0, s_0)

Let the Lagrangian multipliers for constraints (1) and (2) be θ and $\beta^t\pi\left(s^t\right)\mu_t\left(s^t\right)$ respectively. Cumulative multipliers[3], $\zeta_t\left(s^t\right)$, can be defined to make the problem recursive:

$$\zeta\left(s^t\right)=1+\sum_{s^r\preceq s^t}\mu_r\left(s^r\right),$$

where s^r is a subsequent history of s^t. Rewriting cumulative multipliers recursively yields:

$$\begin{aligned}\zeta_t\left(s^t\right) &= \zeta_{t-1}\left(s^{t-1}\right)+\mu_t\left(s^t\right),\\ \zeta_0\left(s_0\right) &= 1\end{aligned}$$

Note that $\{\zeta_t\left(s^t\right)\}$ is a non-decreasing stochastic process.

The Lagrangian can be written as:

$$\begin{aligned}L\left(a_0,s_0,\{\overline{w}_t\}_{t=0}^{\infty},\{R_t\}_{t=0}^{\infty}\right) &= \min_{\theta,\xi_t}\max_{c_t}\sum_t\sum_{s^t}\beta^t\pi\left(s^t\right)\left[\zeta_t\left(a_0,s^t\right)u\left(c_t\right)+\left(1-\zeta_t\left(a_0,s^t\right)\right)U_{aut}\left(\overline{w}_t,s_t\right)\right]\\ &\quad+\theta\left(a_0,s^t\right)\left[\sum_t\sum_{s^t}\prod_{s=1}^{t}\frac{1}{R_{s-1}}\pi\left(s^t\right)\left[y(s_t,\overline{w}_t)-c_t\right]-a_0\right]\end{aligned}$$

[3] Marcet and Marimon (1999)

The first-order condition with respect to c_t is

$$\begin{aligned}\beta^t \zeta_t\left(a_0, s^t\right) u_{c,t} &= \theta\left(a_0, s^t\right) \prod_{s=1}^{t} \frac{1}{R_{s-1}} \\ \beta^t \xi_t\left(a_0, s^t\right) u_{c,t} &= \prod_{s=1}^{t} \frac{1}{R_{s-1}} \end{aligned} \tag{3}$$

where $\xi_t\left(a_0, s^t\right) \equiv \frac{\zeta_t\left(a_0, s^t\right)}{\theta\left(a_0, s^t\right)}$. R_t is the market interest rate faced by all households. ξ_t is a summary statistic of a household's history. It measures how severely and how many times the household has been constrained in history. Therefore, the equation (3) implies that a household's consumption is history dependent, and that a household should have a higher consumption level if it has a higher ξ_t.

The first-order condition implies that the ratio of marginal utilities in consecutive nodes $\left(s^t, s^{t+1}\right)$ satisfies the following restriction:

$$\begin{aligned}\frac{1}{R_t} &= q_t \\ &= \beta \frac{u_{c,t+1}}{u_{c,t}} \frac{\xi_{t+1}}{\xi_t} \\ &= \max_i \beta \frac{u_{c,t+1}}{u_{c,t}}\end{aligned}$$

The intertemporal price, q_t, is equal to the maximum intertemporal marginal rate of substitution (IMRS) across all households. This can be clearly seen by the fact that only an unconstrained household can engage in arbitrage when its IMRS is smaller than the state price of consumption in a particular state of the world. If the price of consumption in that state were larger than IMRS, the household can short a contract that delivers one unit of consumption in that state, or sell a contingent claim, at a price q_t that would increase its overall utility.

In addition, aggregate consumption C_t, aggregate market efficient labor supply L_t, and aggregate home production output H_t can be written as follows:

$$\begin{aligned}C_t &= \sum_{s^t} \int \pi\left(s^t \middle| s_0\right) c_t d\Phi_0 \\ L_t &= \sum_{s^t} \int \pi\left(s^t \middle| s_0\right) s_t l\left(s_t, \overline{w}_t\right) d\Phi_0 \\ H_t &= \sum_{s^t} \int \pi\left(s^t \middle| s_0\right) H\left(1 - l\left(s_t, \overline{w}_t\right)\right) d\Phi_0\end{aligned}$$

The resource constraint for this economy can now be written as

$$C_t + G_t + K_{t+1} = F\left(K_t, L_t\right) + H_t + \left(1 - \delta\right) K_t, \quad for\ \forall t \geq 0.$$

Firms

The firms' problem yields the following first order conditions:

$$\begin{aligned} w_t &= F_{L,t} \\ 1 &= q_t \left[\left(F_{K,t+1} - \delta\right)\left(1 - \tau_{K,t+1}\right) + 1\right] \end{aligned} \tag{4}$$

where $q_t \equiv \frac{1}{R_t}$ is an intertemporal price. Based on these optimal conditions, firms make decisions on labor demand and capital investment.

4.5 Competitive Equilibrium

Definition 4.1 *A competitive equilibrium is a given initial condition $K_{0,}$ a sequence of allocations $\{c_t\left(a_0, s^t\right), l_t, K_t\}$, a sequence of prices $\{w_t, R_t\}$, and a sequence of policies $\{\tau_{L,t}, \tau_{K,t}, B_t, G_t\}$ such that the household problem is solved for each $\left(a_0, s_0\right)$, the firm problem is solved, the government budget constraint is satisfied for all periods and the markets clear.*

5 Ramsey Problem

Given the initial distribution Φ_0 over (a_0, s_0), the government's optimal tax problem is to choose sequences of $\{\overline{w}_t, K_{t+1}, G_t, R_t\}$ consistent with the competitive equilibrium such that social welfare is maximized.[4].

$$\max_{\{\overline{w}_t, K_{t+1}, G_t, R_t\}} \int W_0\left(a_0, s_0, \{\overline{w}_t\}_{t=0}^{\infty}, \{R_t\}_{t=0}^{\infty}\right) d\Phi_0 + \sum_{t=0}^{\infty} \beta^t U(G_t)$$

subject to

$$C_t + K_{t+1} + G_t = F\left(K_t, L_t\right) + H_t + (1 - \delta) K_t \tag{5}$$

Equation (5) is the resource constraint and must hold for all periods. Notice that as is generally the case in Ramsey problems we exclude the government budget constraint, since

[4] The social welfare criterion is assumed to be the equally weighted sum of discounted utilities across population

household consumption choices satisfy the household's budget constraints, which together with the resource constraints imply the government budget constraints.

The Lagrangian is written as

$$\begin{aligned} L_G &= \min_{\lambda_t} \max_{\{\overline{w}_t, K_{t+1}, G_t, R_t\}} \int W_0\left(a_0, s_0, \{\overline{w}_t\}_{t=0}^{\infty}, \{R_t\}_{t=0}^{\infty}\right) d\Phi_0 + \sum_{t=0}^{\infty} \beta^t U(G_t) \\ &\quad + \sum_{t=0}^{\infty} \beta^t \lambda_t \left(F\left(K_t, L_t\right) + H_t + (1-\delta)K_t - C_t - K_{t+1} - G_t\right) \end{aligned}$$

where λ_t is the Lagrange multiplier for the resource constraints.

Then, first order conditions with respect to G_t, K_{t+1} and $\overline{w}_t$ are respectively:

$$U'(G_t) = \lambda_t \tag{6}$$

$$\lambda_t = \beta\lambda_{t+1}(MP_{K,t+1} + 1 - \delta) \tag{7}$$

$$\frac{d\int W_0\left(a_0, s_0, \{\overline{w}_t\}_{t=0}^{\infty}, \{R_t\}_{t=0}^{\infty}\right) d\Phi_0}{d\overline{w}_t} + \frac{d\left[\beta^t \lambda_t \left(F\left(K_t, L_t\right) + H_t - C_t\right)\right]}{d\overline{w}_t} = 0 \tag{8}$$

where $MP_{K,t} = \frac{dF(K_t, L_t)}{dK_t}$.

Moreover, the Envelope Theorem implies

$$\begin{aligned} \int \frac{dW_0\left(a_0, s_0, \{\overline{w}_t\}_{t=0}^{\infty}, \{R_t\}_{t=0}^{\infty}\right)}{d\overline{w}_t} d\Phi_0 &= \underbrace{\int \beta^t \pi\left(s^t\right)(1-\xi_t)\frac{dU_{aut,t}}{d\overline{w}_t} d\Phi_0}_{\text{The benefit of increasing labor tax}} \\ &\quad + \underbrace{\int \beta^t \pi\left(s^t\right)\xi_t u_{c,t} \frac{dy_t}{d\overline{w}_t} d\Phi_0}_{\text{The cost of increasing labor tax}} \end{aligned} \tag{9}$$

6 Steady State Analysis

We define the steady state to one in which all aggregate variables and the distribution of households stays constant; we also assume that the economy converges asymptotically to the steady state. We characterize the steady state allocations in this section and along with this discussion explain how to compute those allocations. We index all of the steady state variables by omitting the subscript t.

6.1 Labor Allocation

Given the steady state after-tax wage rate $\overline{w}$, each household's labor supply $l(s, \overline{w})$ can be decided by a total labor income maximization problem. Then, aggregate labor supply is given by:

$$L = \int l(s, \overline{w}) s d\Phi$$

6.2 Capital Allocation

Proposition 6.1 $\frac{1}{\beta} = MP_K + 1 - \delta$ *in steady state.*

Proof. *The result follows directly from the steady-state version of the following Euler equation in the Ramsey planner's problem.*

$$-U'(G_t) + \beta U'(G_{t+1})(MP_{K,t+1} + 1 - \delta) = 0$$

■

Proposition 1 states that in the steady state pre-tax capital return, $MP_K - \delta$ must equal the rate of time preference, and it characterizes the optimal level of capital stock in the economy.

6.3 Consumption Allocation

Aggregate Consumption Allocation

The aggregate steady state consumption C can be derived from the steady state resource constraint:

$$C = F(K, L) + H - \delta K - G \tag{10}$$

while aggregate consumption is allocated across agents:

$$C = \int c(s) d\Phi$$

Household Consumption Allocation

From equation (3), each household's consumption can be characterized by the linear risk-sharing rule under the assumption of a power utility function with a constant risk-aversion coefficient γ[5]:

$$c_t = \frac{\xi_t^{1/\gamma}}{h_t} C_t$$

where h_t and C_t are defined as

$$\begin{aligned} h_t &\equiv \sum_{s^t} \int \pi\left(s^t\right) \xi_t^{1/\gamma} d\Phi_0 \\ C_t &\equiv \sum_{s^t} \int \pi\left(s^t\right) c_t d\Phi_0 \end{aligned}$$

and

$$u\left(c_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

C_t is the aggregate consumption in period t and h_t is the $1/\gamma$th cross-sectional moment of the cumulative multiplier. Hence, h_t is a non-decreasing process. We interpret h_t as the fraction of the constrained households and the severity of their constraint.

Let $\omega_t \equiv c_t/C_t$ denote the consumption share in period t. The risk-sharing rule implies that the ratio of consumption share between period t and $t+1$ is

$$\frac{\omega_{t+1}}{\omega_t} = \left(\frac{\xi_{t+1}^{1/\gamma}}{\xi_t^{1/\gamma}}\right) g_t^{-1}$$

where $g_t \equiv \frac{h_{t+1}}{h_t}$ is defined as the growth rate of h_t.

When a household does not switch to a state with a binding enforcement constraint (i.e. $\xi_{t+1} \equiv \xi_t + \mu_t = \xi_t$), its consumption share drifts downward at the rate of g_t:

$$\omega_{t+1} = \omega_t \frac{1}{g_t} \tag{11}$$

Steady State Consumption Cutoff Rule

Following the methodology in Lustig (2005), we now describe the steady state household consumption allocation. To implement this goal, we take the consumption share, rather than the cumulative multipliers ξ_t as a state variable, and introduce cutoff rules to characterize steady state household consumption allocations.

[5]See Lustig (2005) for the derivation of the risk sharing rule.

For a given weight growth $\hat{g}$, we define the continuation value function, $V(\omega, s)$ as:

$$V(\omega, s) = \frac{1}{1-\gamma}\left(\frac{\omega}{\hat{g}}C\right)^{1-\gamma} + \beta \sum_{s'} \pi(s'|\, s)\, V(\omega', s') \tag{12}$$

The cutoff, $\underline{\omega}(s)$ is determined such that the participation constraint binds exactly. That is, $\underline{\omega}(s)$ satisfies

$$V(\underline{\omega}(s), s) = V_{aut}(s, \overline{w}) \text{ for each } s.$$

The cutoff values are the consumption shares for agents who switch to a state with a binding constraint, to prevent those agents from defaulting.

In brief, if the constraint of the household does not bind, ω drifts down by the rate of g, as shown by equation (11). If it does bind, ω must equals the cutoff. Hence, the cutoff rule can be shown as follows. For a given weight growth $\hat{g}$, the cutoff rule for the consumption weight is given by:

$$\begin{aligned} \omega' &= \omega \text{ if } \omega > \underline{\omega}(s) \\ \omega' &= \underline{\omega}(s) \text{ if } \omega \leq \underline{\omega}(s) \end{aligned}$$

and the actual individual consumption is given by:

$$c = \frac{\omega'}{\hat{g}}C$$

Therefore, the cutoff rule implies the following. As long as an agent's consumption share is larger than the cutoff value, the agent's actual consumption drifts down at rate g. It keeps shrinking until the agent has a good enough shock realization, such that corresponding cutoff is greater than the previous period's consumption share. If this happens, then the agent's consumption share equals to the cutoff divided by the growth rate of h.

The beauty of the cutoff rule is that, as long as g and C are known, we can characterize each household's dynamic consumption allocation over time. As we shall show later, the computational methodology of solving allocations and prices are based on the cutoff rule.

6.3.1 Optimal Labor Tax Rate

We can derive equation (13) by imposing steady state on equation (8). Thus, the after-tax wage rate in steady state is chosen such that equation (13) is satisfied.

$$\int \omega^{\gamma} \frac{dV_{aut}}{d\overline{w}} d\Phi = \int \omega^{\gamma} u_c \frac{dy}{d\overline{w}} d\Phi \tag{13}$$

If the government increases the steady state after-tax wage rate at margin, then the benefit of increasing after-tax wages in terms of utility is $u_c \frac{dy}{d\overline{w}}$. On the other hand, increasing the after-tax wage rate tightens the enforcement constraint for each household by increasing autarky value by $\frac{dV_{aut}}{d\overline{w}}$. From the Ramsey planner's point of view, the aggregate benefit and cost are the integrated weighted sum of utilities across households, and should be equal in equilibrium, as shown in equation (13). Hence, the optimal after-tax wage rate can be obtained.[6]

6.3.2 Equilibrium Consumption Price

The intertemporal price is

$$\begin{aligned} q_t &= \frac{1}{R_t} \\ &= \beta \left[\frac{C_{t+1}}{C_t} \right]^{-\gamma} g_t^{\gamma} \end{aligned} \tag{14}$$

The first part of equation (14) is a standard shadow price that can be obtained in a representative agent economy. The second part is the multiplicative adjustment of the aggregate shadow cost of the enforcement constraints. The growth rate of h_t, g_t is greater than or equal to one, because h_t is a non-decreasing sequence. Therefore, the risk of being constrained in the future makes one unit of future consumption expensive.

Proposition 6.2 *$\beta < q$ in steady state, if and only if full-risk sharing is not feasible.*
Proof. *In steady state, the average growth rate of consumption is constant at* 1 *and*

$$q = \frac{Q'}{Q} = \beta g^{\gamma} \tag{15}$$

where g is a growth rate of h and greater than 1 *if and only if any positive fraction of households is constrained.* ■

We know that q is an unconstrained household's IMRS and the growth rate of the constrained household's consumption is higher than one. Then, it must be the case that the growth rate of the unconstrained household's consumption is lower than one, to keep the average growth rate constant (i.e. equal to one).Thus, the time discount factor is smaller than the intertemporal price. Note that the steady state spot price of one unit of consumption in the next period is decided by g.

[6] Because $\overline{w} = (1 - \tau_L)w$

7 Optimal Capital Tax

This section explains how we compute the steady state capital tax rate. Proposition 1 provides the planner's Euler equation, based on which the planner chooses the optimal level of capital. Equation (4) is the firm's Euler equation, based on which firms make the capital investment decisions in competitive equilibrium.

We choose the steady state capital tax rate such that these two equations are consistent and equivalent to each other.

$$\begin{aligned} 1 &= q\left[\left(1-\tau_K\right)\left(MP_K-\delta\right)+1\right] \\ &\equiv \beta\left[MP_K+1-\delta\right] \end{aligned}$$

For the private sector to achieve the optimal capital level in competitive equilibrium, the optimal capital tax rate should be the following:

$$\tau_K = 1 - \frac{1/q-1}{1/\beta-1} \tag{16}$$

Proposition 7.1 *Positive capital tax is optimal if and only if $\beta < q$ in steady state.*
Proof. *The result follows directly from Equation* (16) ■

We know that the intertemporal price should be the unconstrained household's intertemporal marginal rate of substitution and the average growth rate of consumption is one in steady state. Given that, the fact that the constrained household's consumption growth is greater than one implies that the unconstrained's consumption growth should be less than one. Therefore, the unconstrained intertemporal marginal rate of substitution should be higher than the time discount factor and so should the intertemporal price.

Recall the dual approach in the Ramsey problem. The government chooses the after-tax rate and the interest rate. As long as the government chooses an interest rate that is consistent with the competitive equilibrium, it will always result in the modified golden rule in steady state, regardless of the frictions in competitive equilibrium.

We argue that this Ramsey problem yields the same outcome (16) of positive capital taxation as long as the market interest rate is lower than time preference rate in steady state.

8 Numerical Example

8.1 Parameters and Shock Process

We compute stationary allocations for a set of parameters, then simulate data from the model to arrive at the model's quantitative prediction. To generate simulated results, we first must parameterize the idiosyncratic labor shock process and preferences. Krueger (1999) uses Storesletten, Telmer and Yaron (2004)'s estimation for the idiosyncratic endowment shock process. They use labor market earnings to calibrate the process for the labor efficiency units from the PSID $(1969-1992)$. Their idiosyncratic process includes the effects of government programs that are devised to share risk, such as unemployment insurance. Since we are interested in income risk, which has to be insured by private arrangements, net of those risks already insured by the government, their idiosyncratic process is actually more appropriate for our study than that of Krueger (1999).

Let z_{it} be the logarithm of individual income. Storesletten, Telmer and Yaron (2004) assume that idiosyncratic income has a persistent and a transitory component and estimate

$$\begin{aligned} z_{it} &= \alpha_i + u_{it} + \varepsilon_{it}, \quad \alpha_{it} \sim Niid\left(0, \sigma_\alpha^2\right) \text{ and } \varepsilon_{it} \sim Niid\left(0, \sigma_\varepsilon^2\right) \\ u_{it} &= \rho u_{it-1} + \eta_{it}, \quad \eta_{it} \sim Niid\left(0, \sigma_\eta^2\right) \end{aligned}$$

They find estimates of $\left(\rho, \sigma_\alpha^2, \sigma_\varepsilon^2, \sigma_\eta^2\right) = (0.98, 0.326, 0.005, 0.019)$. Note that this idiosyncratic process displays a high degree of persistence. Given their estimation, Krueger (1999) ignores the individual-specific fixed effects α_i and approximates the continuous $AR(1)$ process by a 5 state Markov chain, using the procedure described by Tauchen and Hussey (1992). In the following table, we use the baseline parameterization for the idiosyncratic process that Krueger (1999) constructs.

Table I

Krueger (1999) Calibration

Panel A: Markov Chain States
$S = \begin{bmatrix} 0.63 & 0.79 & 0.96 & 1.17 & 1.47 \end{bmatrix}$
Panel B: Stationary Distribution
$\Pi = \begin{bmatrix} 0.183 & 0.212 & 0.210 & 0.212 & 0.183 \end{bmatrix}$
Panel C: Transition Probability
$\pi = \begin{bmatrix} 0.71 & 0.26 & 0.02 & 0.01 & 0.00 \\ 0.23 & 0.51 & 0.24 & 0.02 & 0.00 \\ 0.02 & 0.24 & 0.48 & 0.24 & 0.02 \\ 0.00 & 0.02 & 0.24 & 0.51 & 0.23 \\ 0.00 & 0.01 & 0.02 & 0.26 & 0.71 \end{bmatrix}$

A set of parameters is also summarized in the following table:

Parameters		Value
β	Discount Factor	0.9
γ	Risk Aversion	2.00
δ	Depreciation Rate	0.08
α	Capital Income Share	0.20
λ	Labor Supply Elasticity	1

We set the capital income share α to 0.2 which implies that we set the labor income share to 0.8. Lustig (2004) argues that the average labor income share $(1-\alpha)$ of national income in the US between 1946 and 1999 is 70 percent (source, NIPA). The additional 11 percent is proprietor's income derived from farms and partnerships (primarily doctors and lawyers) and should be treated as labor income for the purpose of this exercise. This brings the total labor income share to around 81 percent.

The specific home production function we used for computation is as follows:

$$H(n) = \frac{1}{D^{\frac{1}{\lambda}}\left(1+\frac{1}{\lambda}\right)}(1 - n^{1+\frac{1}{\lambda}})$$

where D is a constant. By solving the total labor income maximization problem, we can obtain the household market labor supply function

$$n = D\left(\overline{w}s\right)^{\lambda}$$

Hence, λ represents labor supply elasticity. In addition, the parameter D in the home production function is chosen such that when $\tau_l = 0.35$, the average working time per household $\int D\left(\overline{w}s\right)^{\lambda} d\Phi = 1/3$.

8.2 Results

The optimal labor tax and capital tax rates are 16.61 percent and 7.07 percent respectively. Table II and III show the optimal allocations and prices in steady state.

Table II Quantities

$C = 0.9755$	$L = 0.4625$
$K = 0.5539$	$H = 0.6349$
$Y = 0.4812$	$B = -0.2860$

Table III Prices

$w = 0.6768$	$r = 0.1033$
$q = 0.9064$	$R = 1.1033$
$1/\beta = 1.1111$	$g = 1.0036$

9 Conclusion

We study optimal long-run capital taxation in an economy with idiosyncratic labor productivity shocks and enforcement constraints. We prove that a positive capital tax is optimal as long as a positive fraction of the population is constrained; moreover we show that it is the difference between the Ramsey planner's and the private sector's discount factor that motivates imposing the positive capital tax rate.

This paper provides a generalized framework under which taxing capital income is optimal: any endogenous discrepancy between the government's discount factor and that of the private sector will support such a result. On this basis, several extensions make themselves apparent. For example, any model economy in which the private sector faces a borrowing constraint of any kind[7], while the government does not, would support positive capital taxation. We can also consider a model with segmented markets. Such an environment yields

[7] For example, exogenous borrowing constraints in Aiyagari (1995)

positive capital taxation since the private sector's discount factor is greater than the planner's: a positive probability of not being able to participate in future asset markets increases the private sector's discount factor.

In this paper, we have a government that chooses government consumption and aggregate capital level and this allow us to come up with a modified golden rule, while our subsequent paper, Chien and Lee (2005), has the government play a minimal role in the economy: it simply collects tax revenues and transfers it back to households in a lump-sump fashion. In other respects, the environment is the same as in this paper. The subsequent paper solves the planner's problem and decentralizes the constrained efficient allocations by introducing capital taxation. It argues that the optimal capital tax rate should be strictly positive, and should increase over time to a certain point and also shows that the optimal capital tax rate would be higher than one would expect.

Appendix: Computation

For the sake of simplicity, government spending is set at 20 percent of gross market output, i.e., $G/(K^{\alpha}L^{1-\alpha}-\delta K)=0.2$.

Computational Algorithm

The algorithm bases on iterating in τ_L and g.

1. Compute the input ratio from $\frac{1}{\beta}=\alpha(\frac{K}{L})^{\alpha-1}+1-\delta$, and hence obtain w and r.

2. Guess for τ_L,then compute

   - after-tax wage $\overline{w}=(1-\tau_L)\,w$
   - Given $\overline{w}$, compute V_{aut}, L and H.
   - K followed by the input ratio.
   - Compute C by the resource constraint, equation (10).

3. Guess g ($g\equiv\frac{h'}{h}$)

   - Compute the cutoff $\underline{\omega}_s$
   - Simulate the household consumption allocation by cutoff rules

4. Update g by computing the average growth rate of household consumption

5. Iterate (4) and (5) until g converges.

6. Define and compute

$$Dist(\tau_L)=\left|\int\left(\frac{y}{\omega}\right)^{-\gamma}nsd\Phi-C^{-\gamma}L\right|$$

7. Repeat (2) to (6) until $Dist(\tau_L^{*})=0$

8. τ_K is followed by equation (16)

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