Demographic Transition and Industrial Revolution: A Coincidence?

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All industrialized countries experienced a transition from high birth rates and stagnant standards of living to low birth rates and sustained growth in per capita income. What contributed to this transformation? Were output and population dynamics driven by common or separate forces? We develop a general equilibrium model with endogenous fertility in order to quantitatively investigate the English case. We find that mortality decline significantly influences birth rates. Increased productivity has a negligible effect on birth rates but accounts for nearly all of the increase in per capita output, industrialization, urbanization, and the decline of land share in total income.

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I. Introduction

All industrialized countries experienced a transition from stagnant standards of living to sustained growth in per capita income. This transition coincided with the demographic transition from high birth and mortality rates to low birth and mortality rates. Resources reallocated from rural production to non-rural production, and the importance of land's income share in total production significantly declined over the same period of time. These key observations that represent one of the major transformations of modern times motivated this paper.

What factors were responsible for these changes and to what extent? Is there a common explanation for economic and demographic changes, or were output and population dynamics driven by separate forces? These questions are important, especially for sub-Saharan African countries that have not yet undergone the transition to low birth rates. These countries' staggering poverty necessitates effective policy recommendations.

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In order to answer the questions posed above, we develop a dynamic general equilibrium model with endogenous fertility. Within the framework of our model, parameterized to match key moments of 17th century England, we quantitatively assess the importance of two factors in shaping the demographic and economic transformation in England: changes in young-age mortality and technological progress. More precisely, we examine the model dynamics that result when changes in young-age mortality and total factor productivity (TFP) in the rural and non-rural sectors vary over time in accordance with historical data.

We choose to focus on the effect of changes in young-age mortality and TFP because empirical evidence as well as related historical, demographic, and economic literature overwhelmingly point to the existence of an important link between fertility, mortality, and standards of living.

Our model has two important components. First, production is modeled as in Hansen and Prescott (2003). The final good can be produced using two different technologies, the Malthusian, which uses capital, labor, and land as inputs, and, the Solow, which employs capital and labor only. Since land is a fixed factor, it essentially introduces decreasing returns to scale to capital and labor in the Malthusian sector. We associate the Malthusian technology with rural production and the Solow technology with urban production. Our two-technology framework allows us to investigate the implications of changes in young-age mortality and TFP for resource allocation between the two technologies. In this paper we refer to the fraction of non-rural output in total output as the level of industrialization and the fraction of labor employed by the non-rural sector in total labor as the level of urbanization.

The second important component of our mechanism is endogenous fertility. As in Barro and Becker (1989), we assume that parents place value on both the number of surviving children and their children's well-being. Thus, there is a quantity-quality trade-off explicit in our model. Parents face a trade-off between having many children with small inheritance in the form of capital and land for each child and having a few children but endowing each with a larger piece of land and more capital.

How do changes in young-age mortality and TFP propagate in our model? We highlight a few effects here, in particular, the effect of these changes on birth rates and the level of industrialization. There are two channels through which changes in young-age mortality influence households' fertility choices. On one hand, with a higher number of children surviving to adulthood, fewer births are needed to achieve the desired number of surviving children. On the other hand, since both surviving and non-surviving children require parents' time, as the probability of survival increases, the cost of raising a surviving child declines, and hence, induces higher birth rates. Both channels have been emphasized in previous literature. For a useful review, see Wolpin (1997). We find that the fall in young-age mortality represents an important force behind the demographic change in England, accounting for over half of the fall in birth rates.

Our model allows for several channels through which a switch to a faster growing TFP and hence income may alter households' fertility choices. On one hand, children are normal goods, and hence, higher income growth induces higher fertility. On the other hand, rearing children takes time, and with faster growing TFP, the opportunity cost of raising children measured in terms of foregone wage earnings also grows faster dampening fertility. Moreover, with faster rising incomes, parents choose to have higher quality children, which further increases the cost of rearing children.

The view that technological progress governs fertility choices through one or both of the two latter channels is common among historians, demographers and economists. (See Becker and Lewis (1973), Willis (1973), Becker (1981), Hotz, Klerman and Willis (1997), Galor and Weil (2000), Greenwood and Seshadri (2002), and Hansen and Prescott (2003).) In fact, our findings qualitatively agree with this view; changing the growth rate of TFP in the two sectors according to our estimates leads to a decline in birth rates. We, however, find this effect to be quantitatively small.

Interestingly, both acceleration of the non-rural TFP growth and decline in young-age mortality generate full resource reallocation towards the non-rural sector. As the Solow TFP begins to grow faster than the Malthusian TFP, the Solow sector attracts a higher proportion of resources. The result that falling young-age mortality alone, that is, with productivity growth rates held constant, increases the level of industrialization is less intuitive. As the probability of survival increases, the time cost of raising a surviving child declines, augmenting the aggregate labor supply. This results in the relative expansion of output in the Solow sector, which uses labor intensively (Rybczynski-type logic). Although both changes generate a transition from Malthus to Solow, we find that only cahnges in the TFP growth rates are quantitatively relevant for the process of the English industrialization and urbanization, driving the share of the Malthusian output to nearly zero in the period from 1600 to 2000. Changes in the probability of survival lead to a much slower transition, predicting that even in 2400, the output produced by the Malthusian technology would comprise as much as 10 percent of total output.

To summarize the main results, the decline in young-age mortality accounts for 59% of the fall in the Crude Birth Rate[1] that occurred in England between 1650 and 1950. Over the same period, changes in productivity account for 73% of the increase in GDP per capita and for 92% of the decline of land share in total income. Although both experiments generate a transition from Malthus to Solow, changes in TFP do so in a manner consistent with empirical observations. Our finding that changes in TFP alone can account for long term trends in the observed patterns of factor income shares is due to resource reallocation between sectors with different but constant factor elasticities.

Our results suggest that the explanations for economic and demographic changes need not be entirely common. In fact, we find that changes in productivity are quantitatively insignificant in accounting for the observed patterns in fertility behavior, while mortality changes are quantitatively relevant only to population dynamics, and not to the other quantities predicted by the model. This finding does not rule out the possibility that there are important interactions between the two changes treated as exogenously given here, or that some third force is responsible for both changes in technological progress and young-age mortality. Instead, our findings merely suggest that the quantitatively relevant channels through which the demographic and economic transformations transpired are different.

[1] Crude Birth Rate (Crude Death Rate) is the number of births (deaths) in a given year per 1000 people.

The trend in related literature has been making the necessary assumptions on functional forms and parameters in order to guarantee the desired behavior of the dynamical system[2]. The outcome is that there exists a number of insightful dynamical systems capturing potentially very important mechanisms at work; but due to difficulties associated with mapping of these systems to the data, the relative importance of these mechanisms remains unclear. For example, Greenwood and Seshadri (2000), Jones (2001), Kalemli-Ozcan (2002) and Soares (2005) each generate a drop in fertility as well as a take-off to sustained growth regime through the acceleration of technological progress, institutional change, a decline in young-age mortality, and a decline in adult mortality respectively.

The most important contribution of our work is thorough quantitative analysis of young-age mortality and changes in TFP within a dynamic general equilibrium framework with two sectors of production and explicit reproduction choice. The advantage of our approach is that our theory is guided by the available measures of historical data. We use functional forms that allow for straightforward mapping of the model to the data consistent with national income and product accounting[3]; our choice of parameters is restricted by the available data observations in the beginning of the 17th century, and not by the entire time paths of any of the variables in question. The time series used in the design of our experiments represent their actual historical estimates. We work with mortality and fertility data provided by Wrigley, Davies, Oeppen, and Schofield (1997), Mitchell (1978) and the Human Mortality Database. We estimate TFP in the rural and urban sectors using the dual-approach, which requires time series data on wages in the two sectors, land and capital rental rates, and the GDP deflator. These time series were either taken directly or inferred from Clark (2001a, 2001b, 2002)[4]. To sum up, our work calls for a closer interaction of theory and measurement in this branch of literature.

Another important contribution is our analysis of transitional dynamics from one type of balanced growth path towards another, triggered by the observed changes in mortality rates and/or relative TFP growth rates. In earlier works, the prevalent analysis of the exogenous changes was performed by comparing steady states due to the difficulties associated with solving for equilibrium paths in this type of non-stationary environment. We find that a great deal of insight can be lost by leaving the transition path out of consideration as convergence from one steady state to another may take thousands of years.

This paper also contributes to the part of the growth literature that attempts to explain economic development over long time scales. Lucas (2002) emphasizes the importance of this line of research: "... I think it is accurate to say that we have not one but two theories of production: one consistent with the main features of the world economy prior to the industrial revolution [Malthusian theory] and another roughly consistent with the behavior of the advanced economies today

[2]For example, the utility function and its parameters can be *chosen* to guarantee that birth rates fall as income rises.

[3]For example, it is possible to use the available data to compute productivity changes for the production functions in our model. In contrast, for the agricultural good production function assumed in Greenwood and Seshadri (2000) with skilled labor, unskilled labor, and capital as inputs, productivity changes are difficult to measure.

[4]For a complete list of the data sources used, see the Appendix.

[Solow growth theory]. What we need is an understanding of the transition."

The rest of the paper is organized as follows. Section II reviews the English case data and elucidates the up to date accomplishments in related literature. In Section III we set up the model and discuss some equilibrium properties. In Section IV we calibrate the model and in Section V we discuss the solution method. Section VI is a report of main simulation results. In Section VII we perform some sensitivity analysis and we conclude in Section VIII.

II. Some Motivating Facts about England and Wales

We choose to focus on England and Wales due to data limitations elsewhere. Floud and Johnson (2004) and Chesnais (1992) describe England during this period. Galor (2005) provides stylized facts of development.

Figure 1 reports the natural log of an index of real GDP per capita for England and Wales[5]. Observe that per capita real GDP is roughly stagnant for centuries, but takes off in the beginning of the 19th century.

This period is also associated with the trend of people moving out of the rural sector and into the urban sector. As depicted in Figures 2 and 3, the share of the urban GDP rose from around 30% in the 1550s to roughly 98% in the 1990s, while the share of employment in non-rural production increased from around 40% to 98%[6]. Further, Figure 4 illustrates land's income share, which declines from as much as 30% at the onset of the 17th century to nearly 0% today, a remarkable decline in the importance of land as a factor of production.

This major economic transformation was accompanied by remarkable demographic changes (Figure 5)[7]. Before the mid 18th century, both birth and death rates remained high. Average population growth in the first half of the 18th century was low at around 0.4% per year. In the second half of the 18th century, CDR started its fall mainly due to declining adult mortality. Mortality rates in age groups 5-10, 10-15, and 15-25 began their sustained declines in mid 19th century, while younger age mortality 0-5 followed three decades later. (See Wrigley, Davies, Oeppen, and Schofield (1997)). Major factors behind the decline in mortality were the sanitary revolution that reduced fatalities due to water and food borne desease and advances in medical science, most notably, the discovery of benefits of pasteurization, hospitalization, isolation of tuberculosis sufferers, and small pox vaccination.

A sustained fall in birth rates, driven by a fall in marital fertility, occurred from 1870 to 1930, after which both birth and death rates stabilized at their new low levels. Previous changes in birth rates resulted from changes in the timing and incidence of marriage (See Floud and Johnson (2004), Wilson and Woods (1991), and Coale and Treadway (1986)). General Fertility Rate[8], a measure

[5] The data sources are Clark (2001b) for the period from 1560 to 1860 and Angus Maddison (1995) from 1850 to 1992.

[6] The data on the level of industrialization and urbanization up to 1860 are taken from Clark (2001b, 2002); the time series are continued using Maddison's data (1995).

[7] Crude Birth Rates (CBR) and Crude Death Rates (CDR) are taken from Wrigley, Davies, Oeppen, and Schofield (1997) for the time period up to 1871 and continued using the data in Mitchell (1978).

[8] General Fertility Rate is the number of births in a given year per 1000 females of ages 15-44.

less sensitive to the age structure of population than CBR, exhibits similar behavior (Figure 13). Although the fall in birth rates lagged behind the onset of the fall in death rates, it coincided with the fall in young-age mortality (Figure 6). The probability of surviving to the age of 25 is calculated from age-specific mortality rates taken from Wrigley, Davies, Oeppen, and Schofield (1997) and the Human Mortality Database. We use this series to study the effects of changes in young-age mortality. Notice that the lag between the drop in death rates and the drop in birth rates resulted in a hump-shaped population growth rate.

Figure 7 reports our estimates of TFP in the rural and urban sectors. We describe our TFP estimation procedure in Section V. According to our estimates, industrial TFP changed its growth trend earlier than its rural counterpart and on a much larger scale. We use these constant growth trends in TFP to investigate the importance of technological progress in driving the economic and demographic transformations.

III. Related Literature

Several historical and cross-country studies point to young-age mortality as an important determinant of fertility. For a survey of theoretical and empirical work, see Wolpin (1997). Empirical results pointing to mortality as one of the most important determinants of fertility and/or the onset and speed of its decline are reported in Bulatao and Elwan (1985), Woods (1987), Bos and Bulatao (1990) and Karen Mason (1997a) among others[9]. Wolpin (1997) and Shultz (1997) point out that the positive correlation between mortality and fertility is a phenomenon common to both historical transitions of Western Europe and developing countries today.

For theoretical and quantitative studies of the relationship between mortality and fertility, see Ehrlich and Lui (1991), Sah(1991), Wolpin (1997), Eckstein, Mira, and Wolpin (1999), Kalemli-Ozcan, Ryder, and Weil (2000), Kalemli-Ozcan(2002), Lagerloff (2003), Doepke (2004b), Tamura (2005), Soares(2005), Ehrlich and Kim (2005). All of these works embrace a quality-quantity trade-off: parents derive utility from the number of their surviving children (hence, the target number of children theory is incorporated in these frameworks) as well as some measure of their children's average wellbeing[10] that they can alter through resource reallocation[11]. Ehrlich and Lui, Ehrlich and Kim, Kalemli-Ozcan, Lagerloff, Tamura, and Soares explicitly model human capital accumulation and assume increasing returns to scale to parents' human capital and time spent with children in production of children's human capital[12]. Since production of surviving children's

[9] Along with young-age mortality, another significant determinant of fertility found by virtually all empirical studies of developing countries is female education (See Jejeebhoy (1995), United Nations (1995), Bongaarts and Watkins (1996)). However, it appears that female education also strongly affects mortality, suggesting that its effect on fertility may work through its effect on mortality (See Cleland (1990) and Mason (1997b)). Indeed, Schultz (1997), working with low income countries, reports evidence that over half of the total effect of women's education on fertility operates through its indirect effect on child mortality.

[10] In Ehrlich and Lui (1991), the quality-quantity tradeoff arises from parents' desire to ensure old-age support. For another work in which the direction of altruism is reversed and implications of young-age mortality are examined, see Boldrin and Jones (2002).

[11] Doepke also explores a setup in which parents have no control over the average wellbeing of their children.

[12] Doepke also studies a setup with human capital accumulation; he, however, assumes that children's human

human capital requires parents' time in proportion to their fertility[13], a drop in mortality raises the return to human capital investment[14, 15]. The necessary parametric restrictions are then made to ensure a transition to a sustained growth regime through substitutions of quality for quantity and human capital accumulation.

Kalemli-Ozcan and Sah explicitly model the uncertainty of newborns' survival, which gives rise to precautionary motives for having children; a decline in mortality lowers fertility through reducing the hoarding effect. Doepke explores different ways of modeling mortality-fertility interaction, such as discrete fertility choice, stochastic mortality, sequential fertility, and finds that the modeling choice has little bearing on the quantitative results in his setup. Soares and Kalemli-Ozcan et al. investigate the effect of increasing adult longevity on adult human capital accumulation.

A few of the abovementioned works, in particular, Ehrlich and Lui, Kalemli-Ozcan, Lagerloff, Tamura, and Soares, conclude that a decline in child mortality significantly reduces fertility (and the number of surviving children) through its interaction with the quality-quantity tradeoff and pulls the economy onto a sustained growth path. We also incorporate the quality-quantity tradeoff, but quality is measured in units of physical capital and land bequeathed per surviving child. Since both, surviving and non-surviving children, require parents' time, the decline in mortality also relaxes the budget constraint in our model thus allowing parents to optimally respond through the quality-quantity tradeoff. We leave the precautionary demand for children and adult mortality out of consideration and refrain from modeling human capital, which reduces the difficulties of mapping observables into the model. In some sense, our estimates of the effect of mortality decline on fertility represent a lower bound.

Another trend in the literature relates fertility to the growth in income. Hotz, Klerman, and Willis (1997) point out two major reasons for why fertility may be inversely related to income. One is that income elasticity of demand for quality exceeding that of demand for the number of children is simply an empirically plausible assumption (See Becker and Lewis (1973), Bongaarts and Bulatao (2000)). Second is that rearing children takes time; hence, with rising wages, the opportunity cost of time increases (See Wolpin (1998), Becker (1981), Barro and Becker (1988, 1989), Butz and Ward (1979)). Among those who argue that technological progress governed the process of development are Hansen and Prescott (2003), Jeremy Greenwood and Ananth Seshadri (2002), Galor and Weil (2000)[16], and Fernandez-Villaverde (2001).

In Hansen and Prescott (2003), the transition from rural to non-rural production is an equilibrium property of their model brought about by the exogenous technological progress in the Solow technology. The population growth is postulated to be a function of per capita consumption, which

capital is a decreasing returns to scale function of parents' time with children only. In this setup, a decline in child mortality cannot account for a fall in the number of surviving children.

[13]Tamura adjusts for infant mortality.

[14]By relaxing the budget constraint, it allows to produce more surviving children of the same quality and/or endow them with more human capital.

[15]Tamura, Kalemli-Ozcan, Ehrlich and Kim, and Lagerloff model mortality as a function of endogenous variables. Ehrlich and Kim additionally assume that households take that function into account when making choices.

[16]For the quantitative test of the Galor and Weil model, see Lagerlof (2005).

is estimated to match the demographic transition in Europe. For low levels of per capita income, income and population growth are positively correlated and for income levels above a certain threshold, this correlation becomes negative. Our model is closely related to Hansen and Prescott's as we use the same technologies; however, we explicitly model fertility choice as well as young-age mortality.

Greenwood and Seshadri (2000) uses a two sector model with exogenous technological progress and endogenous fertility to study the case of the U.S. The preference parameters are chosen so that with increasing incomes the demand for the agricultural good relative to manufacturing goods declines. Parents substitute quality for quantity since unskilled labor is not an input in production of the manufacturing good. They conclude that changes in TFP alone can account for both, the decline in fertility rates and the increase in GDP per capita that occurred in the U.S. Similarly, Galor and Weil present a novel approach to modeling the process of growth and development. They explicitly model human capital accumulation, endogenous technological change, and fertility. Children's human capital is a function of parents' time with children and the growth rate of TFP, satisfying several assumptions to guarantee that parents' time with children increases in the rate of TFP growth. Hence, parents respond to the acceleration of technological progress by having fewer, higher quality children. The growing stock of human capital feeds back into higher technological progress, thus reenforcing this mechanism. Although technological progress is exogenous in our model, similar channels are present. As TFP accelerates, parents choose to invest a higher fraction of their growing income in quality of their children. Despite the increasing cost per child, however, we find that the quantitative effect of changes in TFP growth rate on fertility is small (in contrast to Lagerloff's findings), although they can account for the increase in GDP per capita, the level of industrialization, urbanization, and factor income shares in England.

The mechanisms proposed by Greenwood and Seshadri and by Galor and Weil, although innovative and insightful, are difficult to test against the data. Preferences and technology parameters as well as TFP growth rates in both models are unidentifiable given the available data measurements[17]. The advantage of our approach is that we use standard technological assumptions, which allows for TFP estimation using the available data. Moreover, the disadvantage of models used by Hansen and Prescott, Greenwood and Seshadri and Galor and Weil is abstraction from mortality. Surviving kids and fertility are represented by the same time series in their models, although these time series behave quite differently over the time period studied. Galor and Weil's model generates a hump-shaped population growth rate without mortality. The observed hump in the population growth rate, however, is due to mortality declining prior to the fertility decline; there is no evidence that fertility exhibited hump-shaped behavior.

Fernandez-Villaverde uses a parameterized framework in which unskilled labor and capital are substitutes while the skilled labor and capital are complements. He feeds in capital-specific technological change that matches the fall in the relative capital equipment price during the years of the

[17] For example, TFP time series estimated by economic historians are usually backed out under the assumption of standard Cobb-Douglas technology and do not account for human capital and/or measures of unskilled v. skilled labor.

falling birth rates: [1875-1920]. This experiment is found important in accounting for the observed patterns of fertility and per capita income in England. A data fact, difficult to reconcile with this force, is that after 1920 the relative price of capital and capital equipment (from the same source used by the author) reverts its downward trend, yet there is no reversion of fertility trend.

Another important quantitative investigation based on a quantity-quality trade-off is Doepke (2004a). Doepke concludes that government policies that impact the opportunity cost of education, such as education subsidies and child-labor laws, have a direct effect on the speed of the demographic transition.

Charles Jones (2001) presents a unique quantitative investigation of the world process of growth and development through a framework which does not incorporate the quality-quantity tradeoff. An assumption on preferences guarantees a fall in birth rates with rising incomes for incomes sufficiently high. Mortality falls as per capita income rises. Since mortality declines before birth rates, the population growth rate exhibits a hump. Technological progress is endogenous due to the presence of the knowledge creating sector and no depreciation of the existing stock of knowledge. The rate of technological progress depends on the size of the population and on the parameter representing the fraction of population employed in the knowledge creating sector. Jones associates this parameter with the advent of institutions promoting innovations and concludes that the timing of the industrial revolution depended crucially on this parameter. He also finds that since 25,000 BC changes in the innovation promoting institutions were more than twice as important as the growing stock of population in accounting for the world's technological progress.

IV. Model

This is a one sector overlapping generations model with two technologies, exogenous technological progress, and endogenous fertility.

Technology, firms

Following Hansen and Prescott (2003), firms are endowed with one of two possible technologies that can produce the consumption good. We subscript the Malthusian technology that requires capital, labor, and land as inputs by 1 and associate it with production taking place in the rural sector. The Solow technology that employs capital and labor as inputs is subscripted by 2 and associated with production taking place in the cities. Both technologies exhibit constant returns to scale, which allows us to assume that there are two aggregate competitive firms, one using the Malthusian technology, and another using the Solow technology. The outputs of these two firms are given by

$$\begin{aligned} Y_{1t} &= A_{1t} K_{1t}^{\phi} L_{1t}^{\mu} \Lambda_t^{1-\phi-\mu}, \\ Y_{2t} &= A_{2t} K_{2t}^{\theta} L_{2t}^{1-\theta}, \end{aligned}$$

where K_j, L_j denote capital and labor employed by technology $j \in \{1, 2\}$, and Λ_t denotes land employed by the Malthusian technology.

We assume exogenous technological progress in both technologies, that is,

$$A_{1t} = A_{10} \prod_{\tau=0}^{t-1} \gamma_{1\tau} \text{ and } A_{2t} = A_{20} \prod_{\tau=0}^{t-1} \gamma_{2\tau},$$

where $\gamma_{i\tau}$ represents the time τ exogenous growth rate of technology i TFP.

Formally, the profit maximization problems of Firms 1 and 2 are given by

$$\max_{K_{1t}, L_{1t}, \Lambda_t} A_{1t} K_{1t}^{\phi} L_{1t}^{\mu} \Lambda_t^{1-\phi-\mu} - w_t L_{1t} - r_t K_{1t} - \rho_t \Lambda_t,$$
$$\max_{K_{2t}, L_{2t}} A_{2t} K_{2t}^{\theta} L_{2t}^{1-\theta} - w_t L_{2t} - r_t K_{2t},$$

where w_t, r_t, and ρ_t denote time t real wage, capital rental rate, and land rental price respectively.

Preferences, households, dynasties

There is measure 1 of identical dynasties, each populated by N_t households at time t. Households live for two periods, childhood and adulthood. An adult household derives utility from its own consumption c_t, the number of its surviving children n_t (young households), and its children's average utility according to[18]

$$U_t = \alpha \log c_t + (1-\alpha) \log n_t + \beta U_{t+1}, \ \alpha, \beta \in (0,1).$$

An adult household rents its land holdings λ_t and capital k_t, and inelastically devotes the time not spent raising children to work. It chooses its own consumption c_t, the number of surviving children n_t, and the amount of bequests k_{t+1} to be passed on to each surviving child in the form of capital, and divides its land holdings equally among its descendants. The time cost of raising each child is q_t. Formally, an adult household's problem is described by the following Bellman equation,

$$\begin{aligned} U(k_t, \lambda_t) &= \max_{c_t, n_t, \lambda_{t+1}, k_{t+1} \geq 0} \alpha \log c_t + (1-\alpha) \log n_t + \beta U(k_{t+1}, \lambda_{t+1}) \\ \text{subject to } c_t + k_{t+1} n_t &= (1 - q_t n_t) w_t + (r_t + 1 - \delta) k_t + \rho_t \lambda_t, \\ \lambda_{t+1} &= \frac{\lambda_t}{n_t}. \end{aligned}$$

Each household takes sequences of wages, capital rental rates, land rental rates, and the time cost of raising children as given and internalizes the effect that his choices today have on the average utility of his descendants. Parents face the quantity-quality trade-off between having many children with small inheritance in the form of capital and land for each child and having a few children but endowing each with a larger piece of land and more capital.

Cost of raising children

A fraction π_t of children born f_t, survives to adulthood. We denote the number of surviving

[18] Notice that the parental utility is increasing and concave in the number of children as in Barro and Becker (1989)'s $U_t = c_t^{\sigma} + \beta n_t^{1-\varepsilon} U_{t+1}$. Further, in the Appendix we prove these preferences are equivalent if $\sigma \to 0$ and $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$. We explore the Barro and Becker utility in the sensitivity section.

descendants by $n_t = \pi_t f_t$.

There is a time cost associated with raising children. A household spends fraction a of its time per each born child and an additional fraction b of its time per each child who survives to adulthood. We assume that for each newborn child, households pay the expected cost of raising him with certainty. Thus, the total time cost of raising f_t newborn children is given by

$$(a + \pi_t b)\, f_t = \left(\frac{a}{\pi_t} + b\right) n_t.$$

Hence, $q_t \equiv \frac{a}{\pi_t} + b$ represents the time cost of raising a surviving child. Observe that the time cost of raising surviving children is a decreasing function of the survival probability. Intuitively, as more newborn children survive to adulthood it becomes cheaper to raise a surviving child [19].

Population dynamics and Market Clearing

The number of adult households evolves according to $N_{t+1} = n_t N_t$.

We use the upper case letters to denote aggregate quantities so that $C_t = c_t N_t,\ K_t = k_t N_t$, $K_{1t} = k_{1t} N_t$, $K_{2t} = k_{2t} N_t$, $L_{1t} = l_{1t} N_t$, $L_{2t} = l_{2t} N_t$. The market clearing conditions in the final goods, capital, labor, and land markets are as follows:

$$\begin{aligned}
C_t + K_{t+1} &= A_{1t} K_{1t}^{\phi} L_{1t}^{\mu} \Lambda_t^{1-\phi-\mu} + A_{2t} K_{2t}^{\theta} L_{2t}^{1-\theta} + (1-\delta) K_t, \\
K_{1t} + K_{2t} &= K_t, \\
L_{1t} + L_{2t} &= (1 - q_t n_t) N_t, \\
\Lambda_t &= \Lambda.
\end{aligned}$$

A. *Equilibrium*

Definition 1 *A competitive equilibrium, for given parameter values and initial conditions* (k_0, N_0)*, consists of allocations* $\{c_t, n_t, \lambda_t, k_{t+1}, k_{1t}, k_{2t}, l_{1t}, l_{2t}, N_{t+1}\}_{t=0}^{\infty}$ *and prices* $\{w_t, r_t, \rho_t\}_{t=0}^{\infty}$ *such that households and firms solve their maximization problems, and all markets clear.*

We next define the Social Planning problem whose solution is the competitive equilibrium allocation. This Social Planning problem (SP) compactly states the optimization problem at hand

[19] If we modeled the cost of raising children to be paid in terms of the final good, the results would not change. In that case, for the existence of a balanced growth path along which per capita variables grow at a positive rate, we would need to assume that the goods cost grows in proportion to income.

and illustrates the sense in which the competitive equilibrium allocation is efficient[20].

$$
\begin{aligned}
& \max_{\{C_t, N_{t+1}, K_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t (\alpha \log C_t + (1-\alpha-\beta) \log N_{t+1}) \\
& \text{s.t.} \\
C_t + K_{t+1} &= F(K_t, L_t; t) + (1-\delta) K_t, \\
L_t &= N_t - q_t N_{t+1}, \\
C_t, K_{t+1}, N_{t+1} &> 0,\ K_0, N_0 \text{ given, where} \\
F(K_t, L_t; t) &= \max_{K_{1t}, L_{1t}} \left[A_{1t} K_{1t}^{\phi} L_{1t}^{\mu} \Lambda^{1-\phi-\mu} + A_{2t} (K_t - K_{1t})^{\theta} (L_t - L_{1t})^{1-\theta} \right] \quad (1) \\
\text{s.t. } 0 &\leq K_{1t} \leq K_t,\ 0 \leq L_{1t} \leq L_t
\end{aligned}
$$

Proposition 2 *The competitive equilibrium in the decentralized economy corresponds to the solution of the Social Planning problem (SP).*

The proof is presented in the Supplemental Notes[21].

Notice that continuity of the objective function in (SP) together with compactness and non-emptiness of the constraint set guarantees existence of a solution. We assume that $1 - \alpha - \beta > 0$ to guarantee that the objective function is strictly concave. Since the constraint set is convex, the solution is unique. In the Supplemental Notes we show that this solution can be characterized using the first order, feasibility, transversality, and optimal resource allocation conditions.

From the Social Planner's perspective, both capital and children are investment goods. By choosing more children today (N_{t+1}), production can be increased tomorrow although at the expense of decreasing production today due to the time cost of raising children. Another interesting tradeoff clear from the setup of the Social Planning problem is the tradeoff between consumption and children today. Indeed, both C_t and N_{t+1} enter the objective function in the Social Planning problem. Hence, children are both consumption and investment goods.

Given K_t and L_t, the Social planner optimally reallocates these resources across the two technologies. Due to decreasing returns to scale in capital and labor, the marginal products of inputs in the Malthusian technology become very large when its capital and labor inputs converge to zero as long as all land is employed. This guarantees that the Malthusian technology is always used in production. The formal proof is included in the Supplemental Notes.

It is instructive to review the intuition that can be obtained from the first order conditions derived from the Social Planning problem:

$$
\begin{aligned}
\frac{C_{t+1}}{C_t} &= \beta (r_{t+1} + 1 - \delta), && (2) \\
\frac{(1-\alpha-\beta) C_t}{\alpha N_{t+1}} &= q_t w_t - \frac{w_{t+1}}{r_{t+1} + 1 - \delta}, && (3) \\
C_t + K_{t+1} &= F(K_t, N_t - q_t N_{t+1}; t) + (1-\delta) K_t, && (4)
\end{aligned}
$$

[20] For a detailed discussion of efficiency in models with endogenous fertility, see Golosov, Jones, Tertilt (2004).
[21] Available through the authors' websites.

where w_t and r_t denote marginal product of labor and capital, $w_t = F_2(K_t, N_t - q_t N_{t+1}; t)$ and $r_t = F_1(K_t, N_t - q_t N_{t+1}; t)$. Equation (2) is a standard Euler equation that describes the intertemporal tradeoff in aggregate consumption. Condition (3) represents the intratemporal tradeoff between consumption and children (N_{t+1}). The marginal rate of substitution between children and consumption is given by their relative price. The price of a child in terms of the final good is measured by the forgone wages due to the time cost of raising children less the present value of the child's earnings in $t+1$. Finally, equation (4) is the feasibility condition.

Limiting Behavior of Equilibrium Time Paths

The behavior of the solution to the model depends on the choice of the parameters and the initial conditions. We can identify three possible types of limiting behavior of the equilibrium time paths: (1) The solution exhibits the property that the relative level of output in the two sectors converges to a constant, (2) The solution exhibits the property that the level of output in the Solow sector converges to 0, (3) The solution exhibits the property that the level of output in the Malthusian sector relative to the total output converges to 0. We refer to these types of limiting behavior of equilibrium time paths as convergence to Malthus-Solow Balanced Growth Path (BGP), Malthus BGP, and Solow BGP respectively[22].

We do not include the discussion of how the choice of parameters and initial conditions affects the limiting behavior of equilibrium time paths in this paper, but a detailed discussion formulated in terms of propositions and proofs is available in the Supplemental Notes. It is, however, useful to point out here that along a Malthus-Solow BGP, both, population growth (n) and per capita output growth (γ), are determined by the TFP growth rates in the two sectors[23]:

$$\gamma = \gamma_2^{\frac{1}{1-\theta}}, \ n = \left(\gamma_1 \gamma_2^{-\frac{1-\phi}{1-\theta}}\right)^{\frac{1}{1-\phi-\mu}}. \tag{5}$$

The growth rate of per capita output increases in the Solow TFP and is independent of the Malthusian TFP. Population growth increases in the Malthusian TFP growth rate and decreases in the Solow TFP growth rate. Interestingly, the time cost of raising children does not enter these two equations. This means that increasing the probability of survival while keeping all other parameters fixed would directly result in the proportional reduction of fertility ($n = \pi f$). For this class of simulations, we found that during the transition from the original to a new BGP, population growth exhibits a hump, and that this transition is lengthy. Therefore, it is misleading to conclude from these comparative statics exercises that mortality changes do not affect population growth. It is important to notice that this analysis is only valid as long as the new value of π does not alter the type of limiting behavior of equilibrium paths, i.e., as long as it does not preclude convergence to a new Malthus-Solow BGP. In fact, in the simulation results of the benchmark economy presented

[22] This discussion contrasts the result obtained by Hansen and Prescott (2003). In Hansen and Prescott, as long as the growth rate of the Solow TFP is positive, all equilibria exhibit convergence to a Solow BGP. In our model, however, the limiting behavior of equilibrium time paths is determined by the particular parameterization as well as the initial conditions.

[23] This result comes from the constancy of the interest rate along a Malthus-Solow BGP and equality of the marginal products of capital in the two sectors. Hence, it is robust to the choice of the objective function.

below, each experiment performed converges to a Solow BGP.

The growth rate of per capita output on a Solow BGP is also given by $\gamma = \gamma_2^{\frac{1}{1-\theta}}$. However, there is no analytical solution for the growth rate of per capita output on a Malthus BGP. The growth rate of population along a Malthus BGP and a Solow BGP also lack analytical solutions. The comparative statics results show that for both of these BGP types, increases in the TFP growth rate lead to a decline in the population growth rate and an increase in per capita output growth rate. For a Malthus BGP, increases in the probability of survival lead to exactly the opposite effect. For a Solow BGP, increases in survival probabilities lead to increases in population growth but do not affect the growth rate of per capita output, $\gamma = \gamma_2^{\frac{1}{1-\theta}}$.

V. Calibration

The objective is to calibrate the parameters of the model to match some key data moments at the beginning of the 17th century England. One important assumption that we make in order to map the data moments into the model parameters is that in the beginning of the 17th century, the economy is on a Malthus-Solow BGP[24].

The data on population growth and mortality rates are available in Wrigley, Davies, Oeppen, and Schofield (1997), Mitchell (1978), and Human Mortality Database. Most other data moments come from Clark's work (2001a, 2001b, 2002). We also need to estimate the time series of TFP in the rural and non-rural sectors. Unfortunately, the data on time series of inputs and outputs of the two sectors, necessary for standard growth accounting, is unavailable. To get around this problem, we implement the dual-approach of TFP estimation, which uses the assumption of profit-maximization. This approach requires time series data on wages in the two sectors, land and capital rental rates, as well as the GDP deflator. These time series we either take directly or infer from Clark (2001b, 2002).

We choose 25 years to represent the length of each time period. The parameters to be calibrated are the Malthusian parameters $A_{10}, \gamma_1, \phi, \mu$, the Solow parameters A_{20}, γ_2, θ, preference parameters α, β, cost of children parameters a, b, π, and the remaining parameters Λ and δ.

Land in the model is a fixed factor whose value we normalize to one ($\Lambda = 1$). Since A_{10} and Λ only enter the model as a product, $A_{10}\Lambda^{1-\phi-\mu}$, we are allowed the second degree of normalization, so we set $A_{10} = 100$. We also set $A_{20} = 100$ as there is no better way to infer it, and sensitivity analysis shows that there is a wide range for A_{20} that will not have any quantitative bearing on the results. It only has the impact on whether the Solow technology is being used in production of output. We have 11 parameters left to calibrate. In order to pin them down we use 11 pieces of information presented in Table 1 below[25].

We simply rewrite the balanced growth path equations in terms of moments and parameters only, then solve for the model parameters using the information about the corresponding moments

[24] Per capita output growth, birth rates, factor shares in total income, young-age mortality, levels of urbanization and industrialization appear stationary during [1580-1650].

[25] Numbers in parenthesis indicate annual rates

in the data. For the description of calibration as a solution to a system of linear equations see the Supplemental Notes.

Table 1: England Around 1600: Data Moments Used for Calibration

Moment	Value	Description
δ	0.723 (0.05)	Depreciation
π	0.67	Probability of survival to 25
$\frac{l_1}{l}$	0.6	Fraction of rural labor in total labor
$\frac{y_1}{y}$	0.67	Fraction of rural output in total output
$\frac{rk}{y}$	0.16	Capital share in total income
$\frac{wl}{y}$	0.6	Labor share in total income
$r+1-\delta$	2.666 (1.04)	Interest rate
qn	0.42	Fraction of time spent with children (or not working)
$\frac{a+b}{a}$	4	Average time cost of surviving children relative to that of non-surviving children
$\gamma_{1,1600}$	1.042 (1.0016)	Growth of rural TFP around 1600
$\gamma_{2,1600}$	1.006 (1.00025)	Growth of non-rural TFP around 1600

Notice that we do not aim to match per capita output growth and population growth in our model because, although stationary, these moments are quite volatile around the beginning of the 17th century. These moments, however, will be compared with their counterparts predicted by the calibrated model. Depreciation and nominal interest rate are reported as 25-year rates, the corresponding annual rates are indicated in parenthesis. Historical estimates of annual depreciation rates range from 2.5% (Clark 2002) to over 15% (Allen 1982). We set $\delta = 0.723$ to match 5% annual depreciation. The probability of surviving to the age of 25 around 1600 was roughly constant at the level of 67%. (See Wrigley, Davies, Oeppen, and Schofield (1997)). Hence, π is also pinned down directly by the data.

Clark (2001b) provides labor and capital shares in total output produced in England as well as relative levels of employment and output in the two sectors. The interest rate is taken from Clark (2001a). The fraction of time spent raising children, qn, is set to 0.42 and will be discussed below. Recall that a is the fraction of time spent on each newborn child while b represents the additional time cost incurred when a child lives to become an adult. We set $\frac{a+b}{a}$ to 4 using an assumption of a linear declining functional form for the instantaneous cost function of raising children in conjunction with the data on young-age mortality rates. See the Appendix for a more detailed explanation of how we arrive at this quantity. The discussion of how $\gamma_{1,1600}$ and $\gamma_{2,1600}$ are obtained follows below.

Calibrating ϕ, μ, θ

We can determine the labor share μ of the Malthusian technology using $\frac{y_1}{y}$, $\frac{l_1}{l}$, $\frac{wl}{y}$ and the equilibrium property that wages equal the marginal product of labor in the Malthusian sector, $w\frac{l}{y} = \left(\frac{\mu y_1}{l_1}\right)\frac{l}{y}$. This implies $\mu = 0.537$.

Now that we know μ, we can pin down the capital share θ of the Solow technology by using $\frac{y_1}{y}$, $\frac{wl}{y}$, and the equilibrium identity that the total labor income is given by the sum of the income paid to the labor employed by the Malthusian technology and labor employed by the Solow technology, $\mu\frac{y_1}{y} + (1-\theta)\frac{y_2}{y} = \frac{wl}{y}$. This determines $\theta = 0.273$.

Similarly, we obtain the capital share ϕ of the Malthusian technology by using $\frac{y_1}{y}$, $\frac{rk}{y}$, and the equilibrium property that the total income paid to capital is the sum of rental income paid to the capital employed in the Malthusian sector and capital employed in the Solow sector, $\phi\frac{y_1}{y} + \theta\frac{y_1}{y} = \frac{rk}{y}$. This gives $\phi = 0.104$.

Calibrating γ_1, γ_2 and estimating TFP time series

We next explain how $\gamma_{1,1600}$ and $\gamma_{2,1600}$ are obtained. We first estimate TFP time series for each sector for the time period of 1585-1915. Then for each sector we fit a trend consisting of two parts each characterized by a constant growth rate. The growth rates characterizing the first part of the TFP trends in the two sectors are denoted by $\gamma_{1,1600}$ and $\gamma_{2,1600}$. In order to estimate the TFP time series, we use the inferred factor income shares in the two sectors, ϕ, μ, θ.

From profit maximization of the firms, using the dual-approach of estimating TFP, we derive

$$A_{1t} = \left(\frac{r_t}{\phi}\right)^{\phi}\left(\frac{w_{1t}}{\mu}\right)^{\mu}\left(\frac{\rho_t}{1-\phi-\mu}\right)^{1-\phi-\mu}, \tag{6}$$

$$A_{2t} = \left(\frac{r_t}{\theta}\right)^{\theta}\left(\frac{w_{2t}}{1-\theta}\right)^{1-\theta}, \tag{7}$$

where r_t (%) is the rental rate on capital, w_t is the real wage measured in units of the final good per unit of labor, and ρ_t is the land rental price measured in units of the final good per acre[26]. Since the data available from Clark is the time series of r_t (%), nominal wages ω_{1t} and ω_{2t} (£), $\tilde{\rho}_t$ (% return on land rents), $P_{\Lambda t}$ (price of land in £/acre), and the GDP deflator P_t, we infer the real wages w_{it} and the real land rental price ρ_t by using

$$w_{it} = \frac{\omega_{it}}{P_t} \text{ and } \rho_t = \frac{\tilde{\rho}_t P_{\Lambda t}}{P_t}.$$

Substituting these into (6) and (7), we obtain the equations that allow us to estimate the Malthusian and Solow TFP time series using the available data:

$$A_{1t} = \left(\frac{r_t}{\phi}\right)^{\phi}\left(\frac{\omega_{1t}}{\mu}\right)^{\mu}\left(\frac{\tilde{\rho}_t P_{\Lambda t}}{1-\phi-\mu}\right)^{1-\phi-\mu} P_t^{\phi-1},$$

$$A_{2t} = \left(\frac{r_t}{\theta}\right)^{\theta}\left(\frac{\omega_{2t}}{1-\theta}\right)^{1-\theta} P_t^{\theta-1}.$$

Figure 7 is a plot of these time series together with the fitted trends. Both, the rural and non-rural TFP time series exhibit a regime switch. Next we explain how we find the two trends.

Let x_t represent the data and y_t its trend, which we restrict to be of the following form:

$$y_t = \begin{cases} y_0 g_1^t & 0 \le t \le \tau \\ y_0 g_1^{\tau} g_2^{t-\tau} & \tau \le t \le T \end{cases},$$

[26] See the Appendix for a more detailed report that would allow anyone to reproduce our TFP estimates.

where g_1 and g_2 denote the growth rates in the first and second growth regimes, and τ represents the timing of the regime switch.

To find the trend we solve

$$\min_{y_0,g_1,g_2,\tau} \sum_{t=0}^{T} (y_t - x_t)^2 .$$

Notice that this procedure determines the two growth rates as well as the timing of the regime switch. Applying this methodology to both of the TFP time series we obtain the trends, presented in Figure 7. That is, we obtain the TFP growth rates characterizing the first part of the trends, $\gamma_{1,1600} = 1.042$ (0.16%) and $\gamma_{2,1600} = 1.006$ (0.025%), as well as the endpoint growth rates $\gamma_{1,1900} = 1.126$ (0.4%) and $\gamma_{2,1900} = 1.174$ (0.6%). The annual percentage growth rates are given in parenthesis. Our estimation results are consistent with Pol Antras and Hans-Joachim Voth (2002) who estimate TFP growth in Britain for the period 1770-1860 using factor prices from sources different from ours and conclude that the TFP growth during this period was no more than 0.6% per year.

Interestingly, $\gamma_{1,1600}$ and $\gamma_{2,1600}$ give prediction to the growth rate of population and per capita output around 1600. Recall that the balanced growth path values for n and γ are determined by γ_1 and γ_2 (See equation (5)). Hence, the obtained values for the growth rates of the Malthusian and Solow TFP imply that $n = \left(\gamma_1 \gamma_2^{-\frac{1-\phi}{1-\theta}}\right)^{\frac{1}{1-\phi-\mu}} = 1.097$ (or 0.37% in annual terms) and $\gamma = \gamma_2^{\frac{1}{1-\theta}} = 1.0085$ (or 0.00034% in annual terms). These predictions are consistent with the data. Indeed, the population in the beginning of the 17th century England grew at the annual rate of 0.4%, while output per capita remained roughly stagnant[27].

Calibrating the remaining parameters

The preference parameter β is given by the Euler equation $\gamma = \frac{\beta}{n}[r + 1 - \delta]$ after we substitute for γ, n, and the gross interest rate. This yields $\beta = 0.415$.

We set the total fraction of time spent raising children qn at 0.42. There is no obvious way to infer qn from the data, but a simple example may be illustrative. Say a person has 100 hours of productive time endowment per week. He works 40 hours, rests 30 hours and spends 30 hours with all of his children. Since there is no leisure in our model, this pattern of time allocation would imply $qn = \frac{30}{30+40} \cong .429$. The sensitivity of results to the choice of qn is addressed in Section VII.

We also set $\frac{a+b}{a} = 4$. Recall that a is the fraction of time spent raising each newborn, and b is the additional cost incurred on surviving children. We pin down fraction $\frac{a+b}{a}$ by assuming the instantaneous cost function of raising a child to be linear and declining with the child's age. We then use data on age-specific mortality rates around 1600 to infer the relative size of b to a. We also perform sensitivity analysis for this fraction and find that the results are very robust to changes in $\frac{a+b}{a}$. Hence, $qn = 0.42$ and $\frac{a+b}{a} = 4$ determine $a = 0.085$ and $b = 0.256$.

The balanced growth path feasibility equation gives prediction for $\frac{c}{k} = r\frac{y}{rk} + 1 - \delta - \gamma n$. Using $\frac{c}{k}, n, \gamma, qn, \frac{l_1}{l}$ along with the data moments, $r, \frac{rk}{y}, \frac{y_1}{y}$, in the remaining balanced growth path equation, $\frac{(1-\alpha-\beta)(1-qn)}{\alpha\mu}\frac{y}{y_1}\frac{1}{r}\frac{rk}{y}\frac{l_1}{l}\rho = qn - \frac{\gamma n}{(r+1-\delta)}$, allows us to calibrate α to 0.582.

[27] We chose not to attempt matching n and γ precisely, because these quantities are highly volatile around 1600.

The calibrated parameters are summarized in Table 2.

Table 2: Summary of Calibrated Parameters

	Value	Description
Malthusian Technology Parameters		
A_{10}	100	Initial level of TFP
$\gamma_{1,1600}$	1.042	TFP growth rate
ϕ	0.104	Capital share
μ	0.537	Labor share
Solow Technology Parameters		
A_{20}	100	Initial level of TFP
$\gamma_{2,1600}$	1.006	TFP growth rate
θ	0.273	Capital share
Preference Parameters		
α	0.582	Weight on consumption
β	0.415	Discount rate
Cost of Children		
a	0.085	Fraction of time spent on each life birth
b	0.256	Additional time spent on each surviving child
Other parameters		
δ	0.723	Depreciation
Λ	1	Land

VI. Simulation Results

Three experiments are conducted within the calibrated framework. The first experiment (Exp 1) is changing the growth rates of TFP in the two sectors according to our estimates arrived at in Section V while keeping young-age mortality at its 1600 level. The second experiment (Exp 2) is changing the probability of surviving to adulthood according to its historical estimates while keeping the growth rates of TFP in both sectors at their 1600 values. The third experiment (Exp 3) is the joint experiment combining the exogenous changes of the first two experiments. The experimental values of γ_1, γ_2 and π are reported in Figures 8-10. Since we do not aim at investigating high frequency behavior, we smooth out[28] the experimental time series.

Each period in the model corresponds to a specific year. The exogenous changes are fed into the model in accordance with their historical estimates; the model is then solved for the equilibrium dynamics under the assumption of perfect foresight[29].

The economy starts off on a Malthus-Solow BGP. Although different types of limiting behavior

[28]The time series for π corresponding to the time period $[1612.5 - 1912.5]$ is replaced by its 7-period moving average. The time series for γ_1 and γ_2 are modified by fitting a logistic function to the endpoint growth rate. The resulting time series of γ_1 and γ_2 corresponding to time periods $[1737.5 - 1837.5]$ and $[1612.5 - 1912.5]$ respectively are replaced by the 5-period moving average.

[29]The solution method is described in detail in the Supplemental Notes.

of equilibrium time paths are possible in our model, all three experiments generate convergence to a Solow BGP, characterized by the Malthusian share of output converging to zero. Figures 11-19 depict the results of the experiments. The dotted lines represent the time paths of relevant variables in the data. The rest of the lines represent the time series of the model counterpart, resulting from each of the experiments. The quantitative results of this experiment for time periods 1600-1950 and 1650-1950 are summarized in Table 3 below.

Table 3: Results

	1600-1950				1650-1950			
		%Accounted by Model				%Accounted by Model		
	%△ in Data	Exp. 1	Exp. 2	Exp. 3	%△ in Data	Exp. 1	Exp. 2	Exp. 3
y	379.6	68.77	2.23	66.21	348.9	73.24	1.68	70
CBR	-48.73	-0.26	44.7	45.84	-39.95	-0.042	59.1	61
GFR	-46.28	-0.59	41.35	44.20	-36.45	-0.10	57	61
$\frac{\rho\Lambda}{y}$	-95.31	92.32	-1.97	91.96	-95.7	91.97	-0.89	92
$\frac{wl}{y}$	16.67	112	-2.39	111.58	20.7	90.22	-0.88	91.13
$\frac{y_2}{y}$	187.88	95.1	-2.03	94.77	177.38	100.72	-1	103.7
$\frac{l_2}{l}$	137.25	98.2	-2.54	97.95	113.26	119.03	-1.46	122.65

In short, we find that the decline in young-age mortality accounts for nearly 60% of the fall in CBR and GFR that occurred in England between 1650 and 1950. Over the same period, changes in productivity account for over 70% of the increase in GDP per capita and nearly all of the decline of land share in total income. Furthermore, we find that changes in productivity are quantitatively insignificant in accounting for the observed patterns in fertility behavior, while mortality changes are quantitatively relevant only to population dynamics, not to the other quantities predicted by the model. The results are described in more detail in what follows.

A. *Experiment 1: Changes in the Growth Rates of TFP*

Recall that the endpoint TFP growth rates are given by $\gamma_{1,1600} = 1.042$ (0.16% annual growth), $\gamma_{2,1600} = 1.006$ (0.025% annual growth), $\gamma_{1,1900} = 1.126$ (0.4% annual growth) and $\gamma_{2,1900} = 1.174$ (0.6% annual growth)[30]. Until the second half of the 18th century, the rural technology enjoyed a somewhat higher TFP growth relative to that of the non-rural technology. Around 1750, the growth rate of the Solow TFP overtook the Malthusian TFP growth. Thus, according to our estimates, the agricultural revolution took place after the industrial revolution and on a smaller scale. It is important to notice that the level of urbanization and industrialization are imperfect data counterparts of l_2/l and y_2/y in our model.

[30] Recall that the time series of TFP growth rates is estimated based on the data up to 1915. For later years, TFP in both sectors is assumed to remain on the same constant growth trends. We investigate the consequences of this assumption in Section VII, where we perform experiments 1 and 3 under the assumption that the Solow TFP accelerated to the growth rate that would yield the growth rate of per capita output in the 20th century.

This experiment generates industrialization in a manner consistent with the data. As TFP in the Solow technology becomes sufficiently large, resources reallocate towards the Solow technology and the fraction of Solow output in total output converges to 1. Observe that changes in TFP growth rates first take place in 1750, hence, this experiment fails to generate any resource reallocation towards the Solow sector prior to 1750.

Labor reallocates from Malthus to Solow in a manner consistent with the data. As the Solow sector becomes continuously more productive relative to the Malthusian sector, it employs a higher fraction of the available resources. The equilibrium path converges to the asymptotic balanced growth path on which the fraction of the Malthusian output relative to total output converges to zero[31].

Acceleration of TFP generates a transition from the Malthusian stagnation to modern growth. Around 1600, the growth rate of per capita GDP is near zero. It then takes off around 1800 and exhibits a sustained growth of nearly 1% per year. In the time period from 1650 to 1950, this experiment accounts for roughly 73% of the increase in per capita GDP in the data.

As resources reallocate towards the Solow sector, the land share in total income declines while the labor share rises. This happens simply because the Solow sector's land share is 0. Notice that factor shares in the two technologies are fixed at the calibrated levels. We conclude that changes in TFP alone can account for long term trends in the observed factor income shares.

Notice from Figures 12 and 13[32] that changes in the TFP growth rates have a very small quantitative impact on fertility behavior. Interestingly, this experiment generates first a rise and then a fall in fertility rates. Recall that the acceleration of productivity and hence income affect birth rates through two different channels in our model. On one hand, children are normal goods, and hence, higher income growth induces higher fertility. On the other hand, rearing children takes time, and with faster growing TFP, the opportunity cost of raising children measured in terms of foregone wage earnings also grows faster dampening fertility. Moreover, with faster rising incomes, parents choose to have higher quality children, which would further increase the cost of rearing children. In fact, if we interpret k_{t+1} as a measure of quality, the ratio k_{t+1}/y_t increases from .0675 to .113. Fertility rises slightly and then declines, overall, the quantitative effect of changes in the TFP growth rates on fertility is very small, which is a very interesting finding.

Comparison of the population growth rate in the data to the one in the model is similar. As depicted in Figure 19, starting at the calibrated level of 0.37% annual rate, population growth first increases, but then decreases converging to 0.36% annual rate in the limit. This experiment generates a small hump in the population growth rate, but it is quantitatively insignificant[33].

[31] It is important to notice that the level of urbanization and industrialization are imperfect data counterparts of l_2/l and y_2/y in our model. The main reason is that we associate the Malthusian sector with rural production and Solow sector with non-rural production. However, in the data rural output is not a perfect substitute of the non-rural output while in the model the Malthusian good is a perfect substitute to the Solow good. It is nonetheless instructive to make these comparisons.

[32] When comparing the results of the experiments to the data in Figures 12-13 and Tables 3-5, we use 3-period moving average representation of CBR and GFR.

[33] As discussed in Section II, the observed hump in the English population growth rate resulted from the fact that CDR fell before CBR. Since we do not model adult mortality, we do not attempt to generate a hump in population

This experiment leads us to conclude that changes in the productivity in the two sectors represent an important force behind the observed patterns in per capita income, the level of industrialization and urbanization, as well as patterns of labor, capital, and land shares in total income. By contrast, changes in productivity are quantitatively unimportant in driving fertility behavior.

The limiting behavior of the equilibrium time paths can be summarized by $y_{t+1}/y_t \to 1.0088$, $\frac{N_{t+1}}{N_t} \to 1.0036$, $r_t \to 1.045$, and $c_t/k_t \to 0.398$, which are given in annualized rates [34].

B. *Experiment 2: Changes in Young-Age Mortality*

The probability of surviving to the age of 25 changed from 67% in 1550 to 98% in 2000. After a slight decline until the onset of the 18th century, the survival probability reversed its trend rising most rapidly only in the 19th century.

Changes in young-age mortality appear to be a major determinant of fertility behavior. When the probability of survival increases, it becomes less costly to produce a surviving child (q declines), which puts upward pressure on fertility. On the other hand, much like in the case of TFP acceleration, as the budget constraint is relaxed due to a drop in q, parents respond by increasing the quality of children: the ratio k_{t+1}/y_t increases from .0675 to .1021. Finally, fertility drops since fewer births are needed to achieve the desired number of surviving children. We find that in the period from 1650 to 1950, changes in the probability of survival roughly account for nearly 60% of the drop in the CBR and GFR that occurred in England.

Figure 16 present the time series of the level of industrialization in the model and in the data using a longer time scale. As the probability of survival declines, the time spent on raising surviving children declines, which frees up time available for work. Then the intuition is similar to the statement of the Rybczynski Theorem from trade, which states that as the endowment of one factor increases, the relative output in the sector that uses that factor intensively also rises. Similarly, as more time becomes available for work, the output in the sector that uses labor more intensively (the Solow sector for our calibration) increases relative to the Malthusian output. In the long run, resources reallocate towards the Solow technology, although it takes a very long time. Even in 2400, as much as 10% of total output is still produced by the Malthusian technology. The model versus data patterns of the level of urbanization look very similar to those of industrialization.

Figure 11 illustrates the result that changes in the probability of survival are quantitatively insignificant in accounting for patterns in GDP per capita. The population growth rate does go up from the annual rate of 0.37% to the annual rate of 0.8%, but the increase is small quantitatively.

We conclude that changes in young-age mortality were an important driving force behind the demographic changes in England but had little bearing on the economic changes that took place in the time period 1650-1950.

growth.

[34] Only one of the eigen values for this dynamical system is less than 1. Notice that if the Malthusian technology does not operate, N_t is no longer a state variable for the rest of the system, which means that the only state variable is k_t and exactly one eigen value needs to be less that 1 for local stability. Hence, the Solow balanced growth path to which the equilibrium time paths converge as a result of changes in TFP growth rates is locally stable.

The limiting behavior of equilibrium time paths can be summarized by $y_{t+1}/y_t \rightarrow 1.00034$, $\frac{N_{t+1}}{N_t} \rightarrow 1.008$, $r_t \rightarrow 1.04$, and $c_t/k_t \rightarrow 0.357$, which are given in annualized rates. Only one of the eigen values for this dynamical system is less than 1. Hence, the Solow balanced growth path to which the equilibrium time paths converge as a result of changes in young-age mortality is locally stable.

C. Experiment 3: Joint Experiment

There appears to be no special interaction between the two exogenous changes studied in this section in the sense that when a joint experiment is performed with both, TFP growth rates and young-age mortality, changing according to their historical estimates, the effects presented here for the two separate experiments essentially add up.

The limiting behavior of equilibrium time paths can be summarized by $y_{t+1}/y_t \rightarrow 1.0088$, $\frac{N_{t+1}}{N_t} \rightarrow 1.008$, $r_t \rightarrow 1.05$, and $c_t/k_t \rightarrow 0.45$, which are given in annualized rates.

VII. Sensitivity Analysis

TFP experiment

Recall that the time series of TFP growth rates were estimated based on the data up to 1910. For later years TFP in both sectors was assumed to remain on the same constant growth trends. What if the growth rate of TFP increased further since 1910? Would changes in TFP growth rates be more successful at accounting for the demographic and economic changes in that case? In this sensitivity exercise, we use a different time series for the Solow TFP, the one that would guarantee that our model captures the growth rate of per capita income in the 20th century. We know that in the limit, convergence takes place to the Solow only BGP. Hence, we can back out γ_2 that would give the limiting $\gamma = \gamma_2^{\frac{1}{1-\theta}} = 1.014$ (the annual growth rate of real GDP per capita). This yields $\gamma_2 = 1.00978$, in contrast to 1.0064 used in our experiments. The table below presents the results of Experiments 1 and 3 modified so that the Solow TFP growth rate beginning with 1912.5 is changed to 1.00978. The original results are shown for comparison.

Table 4: Sensitivity to the Endpoint Solow TFP Growth

	1600-1950				1650-1950			
	%Accounted by Model				%Accounted by Model			
	$\gamma_{2,1900} = 1.0064$		$\gamma_{2,1900} = 1.0098$		$\gamma_{2,1900} = 1.0064$		$\gamma_{2,1900} = 1.0098$	
	Exp. 1	Exp. 3	Exp. 1	Exp. 3	Exp. 1	Exp. 3	Exp. 1	Exp. 3
y	68.77	66.21	82.4	79.48	73.24	70	87.84	83.9
CBR	−0.26	45.84	−0.28	45.87	−0.042	61	−0.07	60.72
GFR	−0.59	44.20	−0.64	44.26	−0.10	61	−0.165	61.10
$\frac{\rho\Lambda}{y}$	92.32	91.96	95.91	95.62	91.97	92	95.55	95.35
$\frac{wl}{y}$	112	111.58	116.4	116.02	90.22	91.13	93.73	94.71
$\frac{y_2}{y}$	95.1	94.77	98.8	98.53	100.72	103.7	104.64	107.8
$\frac{l_2}{l}$	98.2	97.95	101.4	101.21	119.03	122.65	122.91	126.65

The result that changes in the TFP growth rates drive the economic transformation is strengthened. The quantitative effect on birth rates is unaffected.

Barro and Becker Preferences

As proved in the Appendix, the parental utility that we assumed, $U_t(c_t, n_t, U_{t+1}) = \alpha \log c_t + (1-\alpha)\log n_t + \beta U_{t+1}$, is a special case of the Barro and Becker parental utility, $U_t(c_t, n_t, U_{t+1}) = c_t^{\sigma} + \beta n_t^{1-\varepsilon} U_{t+1}$, if $\sigma \to 0$ and $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$. Notice that this implies that $\varepsilon \to 1$. Hence, a natural question is whether our main results remain if we use the Barro and Becker parental utility form with $\sigma > 0$ and $\varepsilon < 1$.

We follow similar steps to recalibrate the model under the assumption of the Barro and Becker utility, except that this calibration procedure does not pin down both ε and σ. Instead, it pins down $\frac{1-\varepsilon-\sigma}{\sigma} = 0.0129$, thus allowing us the choice of ε. We performed the experiments with different values of ε in the admissible range of $(0,1)$. When $\varepsilon = .9$, which implies that $\sigma = 0.0987$, the results are very close to the results reported for the Lucas utility, that is, they are not very sensitive to the choice of ε. Here we report the results for a more extreme case, with $\varepsilon = 0.7$ and the implied $\sigma = 0.2962$.

Table 5: Sensitivity to the choice of preferences

	1600-1950				1650-1950			
		%Accounted by Model				%Accounted by Model		
	%△ in Data	Exp. 1	Exp. 2	Exp. 3	%△ in Data	Exp. 1	Exp. 2	Exp. 3
y	379.6	61.84	1.09	56.17	348.9	66	0.34	59
CBR	−48.73	−8.23	41.01	36.65	−39.95	−9.48	54.26	49.53
GFR	−46.28	−19.85	32.17	20.46	−36.45	−23.8	44.4	31
$\frac{\rho\Lambda}{y}$	−95.31	97.68	−1.46	97.47	−95.7	97.32	0.22	97.2
$\frac{wl}{y}$	16.67	118.51	−1.77	118.3	20.7	95.47	0.22	97.3
$\frac{y_2}{y}$	187.88	100.62	−1.51	100.5	177.38	106.58	0.25	111.9
$\frac{l_2}{l}$	137.25	103	−1.93	102.9	113.26	124.83	0.26	131

In short, the main result, that changes in young-age mortality drive mainly the demographic changes while changes in the growth rates of TFP drive all the other quantities, still remain. The only difference is that with Barro and Becker utility and $\varepsilon = .7$, the effect of mortality on birth rates is just slightly weaker overall, the hump in birth and general fertility rates is more pronounced.

Sensitivity to δ, $(a+b)/a$, *and* qn.

All the quantitative results are extremely robust to changes in δ. Since the estimates of δ vary from 2.5% to 15% in the literature as mentioned above, we investigated δ in this range.

Recall that $(a+b)/a$ is an estimate of the average time cost of surviving children relative to that of non-surviving children. In the Appendix we show in detail how we arrive at the estimate of $(a+b)/a = 4$. It is still fair to say that it is unclear what this fraction should be and hence sensitivity analysis is required here. Notice that this fraction only affects the values of a and b, it does not affect the time cost of raising surviving children q. In particular, a decreases and b increases in $(a+b)/a$. The results are again very robust to the assumptions on $(a+b)/a$. Higher $(a+b)/a$ slightly increases the importance of π in driving the birth rates in England while lower

$(a + b) / a$ slightly decreases the importance of π. Overall, we examined values for $(a + b) / a$ ranging from 1 to 7 and the results were not affected significantly.

Finally, we set the fraction of time spent raising children, qn, to equal .42. Unfortunately, for $qn \leq .411$, we have $1 - \alpha - \beta < 0$, or equivalently $1 - \varepsilon - \sigma < 0$ for the Barro-Becker preferences, which implies that the Planner's utility decreases in the size of population. Although this does not mean that the Planner will set the population size to 0 as households would still be valued as a factor of production, we would not be able to guarantee strict concavity of the objective function. Thus, we do not perform any experiments with a value of qn lower than .42. Raising qn to a higher value does not change the results much. We analyzed the results when qn is as high as .7.

VIII. Conclusion

We develop a general equilibrium model with endogenous fertility in order to quantitatively assess the impact of changes in young-age mortality and technological progress on the demographic transition and industrialization in England. We find that the decline in young-age mortality accounts for nearly 60% of the fall in the General Fertility Rate that occurred in England between 1650 and 1950. Over the same period, changes in productivity account for 73% of the increase in GDP per capita and nearly all of the decline of land share in total income. Interestingly, both experiments generate a transition from Malthus to Solow. However, changes in TFP do so in a manner consistent with empirical observations, driving the share of the Malthusian technology to nearly zero in the period from 1600 to 2000. Changes in the probability of survival generate a much slower transition, predicting that even in 2400 the output produced in the rural sector would still comprise 10% of total output. We also find that changes in TFP alone can account for long term trends in the observed factor income shares. This occurs as a result of resource reallocation between sectors with different factor intensities that remain constant over time.

One of the questions we raised was whether some common forces induced both changes in output and population. Our quantitative results suggest that the explanation for changes in output and population need not be entirely common. In fact, we find that changes in productivity are quantitatively insignificant in accounting for the observed patterns in fertility behavior, while mortality changes are quantitatively relevant only to population dynamics, and not to the other quantities predicted by the model.

An important contribution of this work is thorough quantitative analysis of the equilibrium time paths within the framework that is capable of generating a transition from Malthusian stagnation to modern growth. We feed the exogenous changes into the calibrated framework according to historical data. Every period in our model corresponds to a particular time period in the data. We estimate total factor productivity in the rural and urban sectors using the dual-approach, which utilizes time series data on wages in the two sectors, land and capital rental rates, as well as the GDP deflator. The advantage of our approach is that we use functional forms that allow for direct mapping of observables into the model. Our choice of parameters is restricted by the available data observations in the beginning of the 17th century, and not by the entire time paths of any of the variables in question.

Future work, in our opinion, should be directed at merging this new promising field of unified growth theory with quantitative analysis. In particular, insightful mechanisms such as those put forth by Galor and Weil, Greenwood and Seshadri, Soares, or Kalemli-Ozcan (that provide additional channels through which mortality and technological progress can influence reproductive choices as well as economic growth) need to be brought closer to the data through modification of models as well as innovative approaches to the available data measurements.

Appendix

Data Sources

- Fraction of non-rural labor in total labor, L_2/L

[1565-1865] - Clark, 2001b, Table 1, p. 8
[1820 - 1992] - Maddison, 1995, p. 253

- Index of Real GDP per capita, y

[1565-1865] - Clark, 2001b, Table 7, p. 30, rescaled to match 100 in 1565
[1820-1990] - Maddison, 1995, p. 194, rescaled to match Clark's index in 1850.

- Labor Share in Total Income, wL/Y

[1585 - 1865] - Clark, 2001b, Table 9, p. 46
[1924 - 1973] - Matthews, Feinstein, Odling-Smee, 1982, p. 164
Average for [1973 - 1982] - Maddison, 1987, p. 659
1992 - Gollin, 2002, p. 470, Table 2, Adjustment 3

- Land Share in Total Income, $\rho\Lambda/Y$

[1585 - 1865] - Clark, 2001b, Table 9, p. 46
[1873 - 1913] - Matthews, Feinstein, Odling-Smee, 1982, p. 643
[1987 - 1998] - UK National Statistics

- Capital Share in Total Income, imputed according to $rK/Y = 1 - wL/Y - \rho\Lambda/Y$.

- Fraction of non-rural output in total output, Y_2/Y

[1555-1865] - imputed by dividing Nominal Net Farm Output (alternative labor) obtained from Clark, 2002 (Table 4, p. 14) by Nominal GDP adjusted for missing capital income obtained from Clark, 2001b (Table 3, p. 19).
[1788-1991] - Mitchell, 1978

- Crude Birth and Crude Death Rates

[1541 - 1871] - Wrigley, Davies, Oepenn, and Schofield, 1997
[1871 - 1986] - Mitchell, 1978

- General Fertility Rate

Computed using CBR and fraction of females in total population by
[1541 - 1841] - Wrigley, Davies, Oepenn, and Schofield, 1997
[1841 - 1999] - Human Mortality Database.

- Population Growth Rate

[1541 - 1836] - Wrigley, Davies, Oepenn, and Schofield, 1997
[1841 - 1999] - Human Mortality Database

- Age-specific survival probabilities

[1580-1837] - Wrigley, Davies, Oepenn, and Schofield, 1997
[1841 - 1999] - Human Mortality Database

- Data used for TFP estimation (See the Appendix Section on TFP estimation).

Clark, 2001b
Clark, 2002

Barro and Becker v. Lucas Utility

Proposition 3 *Parental utility used in Lucas (2002),*

$$U_t\left(c_t, n_t, U_{t+1}\right) = \alpha \log c_t + (1-\alpha)\log n_t + \beta U_{t+1},$$

represents the same preferences as represented by the Barro and Becker utility

$$U_t\left(c_t, n_t, U_{t+1}\right) = c_t^{\sigma} + \beta n_t^{1-\varepsilon} U_{t+1}$$

if $\sigma \to 0$ *and* $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$.

Proof. Let $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$. Consider the following transformation of the Barro and Becker utility, $W_t\left(c_t, n_t, U_{t+1}\right) = (1-\beta)\, U_t\left(c_t, n_t, U_{t+1}\right)$,

$$W_t\left(c_t, n_t, W_{t+1}\right) = (1-\beta)\, c_t^{\sigma} + \beta n_t^{1-\varepsilon} W_{t+1}.$$

Next consider another transformation, $V_t\left(c_t, n_t, W_{t+1}\right) = W_t\left(c_t, n_t, W_{t+1}\right)^{\frac{\alpha}{(1-\beta)\sigma}}$,

$$V_t\left(c_t, n_t, V_{t+1}\right) = \left[(1-\beta)\, c_t^{\sigma} + \beta n_t^{1-\varepsilon} V_{t+1}^{\frac{(1-\beta)\sigma}{\alpha}}\right]^{\frac{\alpha}{(1-\beta)\sigma}} = \left(\left[(1-\beta)\, c_t^{\sigma} + \beta \left(n_t^{\frac{1-\varepsilon}{\sigma}} V_{t+1}^{\frac{(1-\beta)}{\alpha}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}\right)^{\frac{\alpha}{(1-\beta)}}.$$

Now taking limits as $\sigma \to 0$ while ε changes so that $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$ we have

$$\lim_{\sigma\to 0} V_t\left(c_t, n_t, V_{t+1}\right) = \left(\lim_{\sigma\to 0}\left[(1-\beta)\, c_t^{\sigma} + \beta \left(n_t^{\frac{1-\varepsilon}{\sigma}} V_{t+1}^{\frac{(1-\beta)}{\alpha}}\right)^{\sigma}\right]^{\frac{1}{\sigma}}\right)^{\frac{\alpha}{(1-\beta)}} = \left(c_t^{1-\beta}\left(n_t^{\frac{1-\varepsilon}{\sigma}} V_{t+1}^{\frac{(1-\beta)}{\alpha}}\right)^{\beta}\right)^{\frac{\alpha}{(1-\beta)}}.$$

Notice that $n_t^{\frac{1-\varepsilon}{\sigma}}$ and $V_{t+1}^{\frac{(1-\beta)}{\alpha}}$ remain fixed as $\sigma \to 0$. Finally, consider another transformation, $U_t(c_t, n_t, V_{t+1}) = \log V_t(c_t, n_t, V_{t+1})$,

$$U_t(c_t, n_t, U_{t+1}) = \frac{\alpha}{(1-\beta)}\left[(1-\beta)\log c_t + \frac{1-\varepsilon}{\sigma}\beta \log n_t + \frac{(1-\beta)}{\alpha}\beta U_{t+1}\right].$$

Simplifying and using the assumption that $\frac{1-\varepsilon-\sigma}{\sigma} = \frac{1-\alpha-\beta}{\alpha\beta}$, i.e., $\frac{1-\varepsilon}{\sigma} = \frac{(1-\alpha)(1-\beta)}{\alpha\beta}$, gives

$$\begin{aligned} U_t(c_t, n_t, U_{t+1}) &= \alpha \log c_t + \frac{\alpha}{(1-\beta)}\frac{(1-\alpha)(1-\beta)}{\alpha\beta}\beta \log n_t + \beta U_{t+1} \\ &= \alpha \log c_t + (1-\alpha)\log n_t + \beta U_{t+1}. \end{aligned}$$

Cost of Raising Children, Measuring $(a+b)/a$

In this Appendix we explain how we arrived at the average time cost of surviving children relative to that of non-surviving children,.$(a+b)/a = 4$, the moment used in calibration. First we compute the average cost of a surviving child, $a+b$. Denoting the momentary cost of raising children by $p(t)$, the total cost of raising a child to the age of τ, $c(\tau)$, is given by $c(\tau) = \int_0^\tau p(t)\,dt$. Under the assumption that the momentary cost is given by a linear decreasing function $p(t) = \eta - \frac{\eta}{25}t$, where η denotes the intercept, we have $c(\tau) = \tau\eta - \frac{\tau^2}{50}\eta$ and the total cost of raising a surviving child becomes $a + b = c(25) = 25\eta - \frac{25^2}{50}\eta = 12.5\eta$.

Next we show how we compute the average cost of raising a non-surviving child, a. Figure 20 illustrates the beginning of the 17th century age specific mortality distribution of those children who died before reaching the age of 25; the age groups are 0-1, 1-5, 5-10, 10-15, and 15-25. The first point, for example, illustrates that out of all children who do not survive to the age of 25, 45% die in the first year of their life. This pattern of age-specific mortality conditional on dying before the age of 25 is persistent throughout the years and looks similar even today. It is important to note this here in order to understand that updating a/b as a part of Experiment 2 and 3 in this paper would not change the results significantly. Assuming the costs are incurred in the middle of the above age groups gives

$$\begin{aligned} a &= 0.45c(0.5) + 0.22c(3) + 0.12c(7.5) + 0.05c(12.5) + 0.16c(20) = 4\eta, \\ b &= 12.5\eta - 4\eta = 8.5\eta. \end{aligned}$$

It follows that $\frac{b}{a} = 2.15$ and $\frac{a+b}{a} = 3.15$.

Assuming the costs are incurred in the beginning of the age groups, gives a higher ratio $\frac{b}{a} = 3.45$, and hence we have $\frac{a+b}{a} = 4.45$. Since most deaths tend to occur early on, we choose $\frac{a+b}{a} = 4$, which corresponds to all age-specific deaths occuring at a fraction $\frac{1}{10}$ of time period lengths corresponding to the age groups.

Estimation of TFP Time Series

Given the calibration for ϕ, μ, θ, we back out time series for A_{1t} and A_{2t} under the assumption of profit maximization,

$$A_{1t} = \left(\frac{r_t}{\phi}\right)^{\phi}\left(\frac{w_t}{\mu}\right)^{\mu}\left(\frac{\rho_t}{1-\phi-\mu}\right),^{1-\phi-\mu}$$
$$A_{2t} = \left(\frac{r_t}{\theta}\right)^{\theta}\left(\frac{w_t}{1-\theta}\right)^{1-\theta}.$$

In the expressions above, r_t is the rental rate of capital (%/100), w_t is the real wage (final goods per unit of labor), ρ_t is the rental price of land (final goods per acre).

We further explain that we work with historical data on r_t (%/100), ω_{1t} (nominal rural wages, £), $\tilde{\rho}_t$ (rental rate of land, %/100), $P_{\Lambda t}$ (price of land, £/acre), and GDP deflator, P_t. These time series may be used to obtain the real wage and the rental price of land through the following identities

$$w_{it} = \frac{\omega_{it}}{P_t},\ \rho_t = \frac{\tilde{\rho}_t P_{\Lambda t}}{P_t}.$$

The GDP deflator, P_t, is obtained from Table 9 in Clark (2001b), and for later time period (1875-1910) it is imputed under the assumption that it grew at the same rate as agricultural prices provided in Table 1 of Clark (2002).

Table 1 in Clark (2002) contains nominal wages in the rural sector ω_{1t} (d, pences per day). Dividing these time series by 240 changes the units into pounds, £. Further, multiplying the resulting time series by 300 gives annual nominal wage ω_{1t} under the assumption that 300 days are worked per year. We infer ω_{2t} using the time series for the wage bill in the rural sector, $\omega_1 L_1$, the total wage bill in the economy $\omega_1 L_1 + \omega_2 L_2$, the fraction of rural labor in total labor $\frac{L_1}{L}$, and the following identity:

$$\frac{\omega_1 L_1 + \omega_2 L_2}{\omega_2 L_2} = \frac{\omega_1 L_1}{\omega_2 L_2} + 1,$$
$$\omega_2 = \frac{\omega_1}{\frac{\omega_1 L_1 + \omega_2 L_2}{\omega_2 L_2} - 1}\frac{1}{\frac{1}{\frac{L_1}{L}} - 1}.$$

The time series of the wage bill in the rural sector, $\omega_1 L_1$ is given in Table 3 of Clark (2002). The total wage bill in the economy $\omega_1 L_1 + \omega_2 L_2$ is taken from Table 3 in Clark (2001b) and for the later period (1875-1910) it is imputed using the time series of $\omega_1 L_1$ and the assumption that the

ratio $\omega_1 L_1/(\omega_1 L_1 + \omega_2 L_2)$ continued to fall at the same rate as between 1865 and 1875. The fraction of rural labor in total labor $\frac{L_1}{L}$ is obtained from Table 1 of Clark (2001b) and for the later period (1875-1910) from Maddison (1995) (page 253).

Having obtained ω_{1t} and ω_{2t}, we back out real wages according to $w_{it} = \frac{\omega_{it}}{P_t}$.

We obtain $\tilde{\rho}_t$ (rental rate of land, %/100) from Table 2 in Clark (2002). Following Clark (2002) (p. 6), we infer $r_t = \tilde{\rho}_t + 0.04$ allowing 1.5% for risk premium and 2.5% for depreciation.

Table 4 in Clark (2002b) provides us with "Total Land Rents and Local Taxes," which represents $\tilde{\rho}_t P_{\Lambda t}\Lambda$, where $P_{\Lambda t}$ is the price of land, £/acre. Dividing this time series by Λ =26.524 M of acres taken from Clark (2002) (p. 10), and by P_t, we obtain $\rho_t = \frac{\tilde{\rho}_t P_{\Lambda t}}{P_t}$.

Mapping of the Model to the Data: Population Size, CBR, GFR

We need to estimate the average size of population in period t. The number of adults is constant at $2N$ over the duration of a period. The number of children changes during each period due to child mortality. At the beginning of the period, $2fN$ children are born. Using age-specific child mortality rates for groups 0-1,1-5,5-10,10-15, and 15-25 and assuming that age-specific deaths occur at a fraction $\nu = \frac{1}{10}$ of time period lengths[35] corresponding to the age groups, we compute the average population size in a particular period according to

$$\begin{aligned} P &= 2N + \left[\left(\nu + (1-\nu)\,\pi_0^1\right) + 4\left(\nu\pi_0^1 + (1-\nu)\,\pi_0^5\right) + 5\left(\nu\pi_0^5 + (1-\nu)\,\pi_0^{10}\right)\right. \\ &\quad \left.+5\left(\nu\pi_0^5 + (1-\nu)\,\pi_0^{10}\right) + 10\left(\nu\pi_0^5 + (1-\nu)\,\pi_0^{25}\right)\right]\frac{1}{25}2fN. \end{aligned}$$

The model counterpart of CBR is then given by $CBR = 1000\frac{2fN}{P}$. Further, GFR, the number of births in a given period per 1000 women of childbearing age is computed according to $GFR = 1000\frac{2fN}{N} = 2000f$. Since, the length of a model time period is 25 years, we adjust the time series of CBR and GFR resulting from the experiments by the factor of $\frac{1}{25}$ when making comparison to the annual CBR and GFR taken from the data.

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[35] For example, all those not surviving to the age of 1 die as 1.2 months old. We choose $\nu = \frac{1}{10}$ because death tends to occur early on. The results, however, are not very sensitive to the choice of ν.

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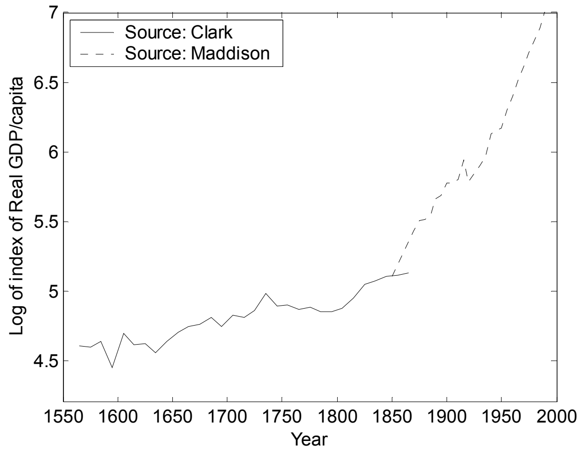

Figure 1. England: ln(real GDP/capita index)

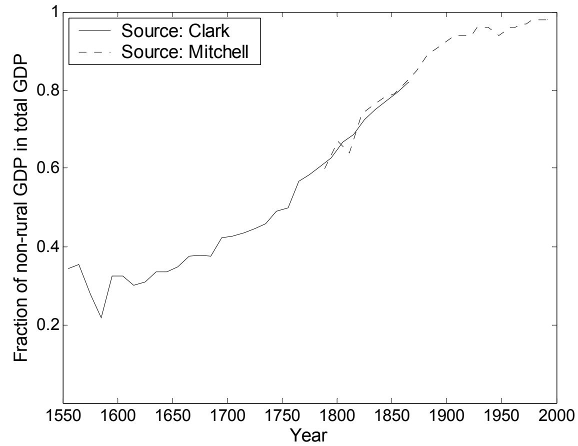

Figure 2. England: Industrialization

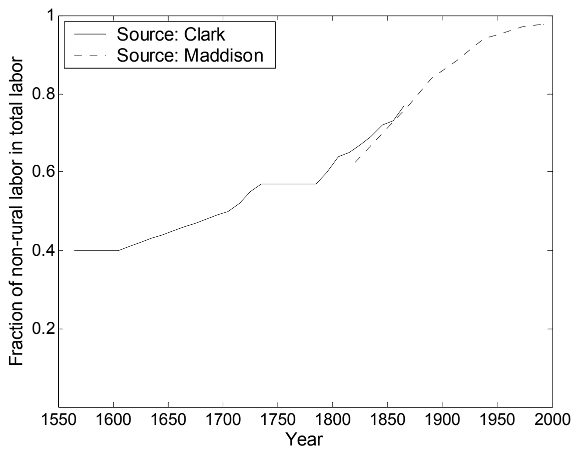

Figure 3. England: Urbanization

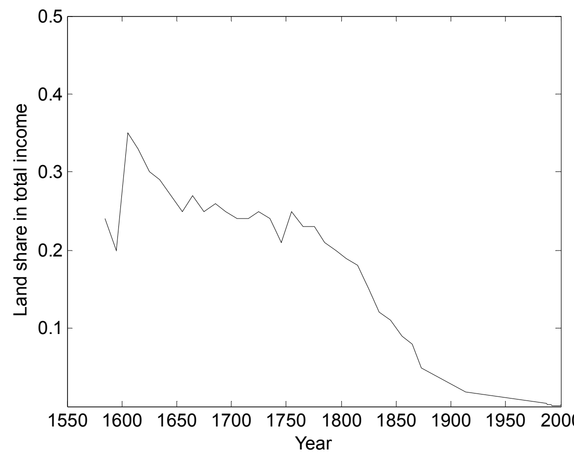

Figure 4. England: Land Share

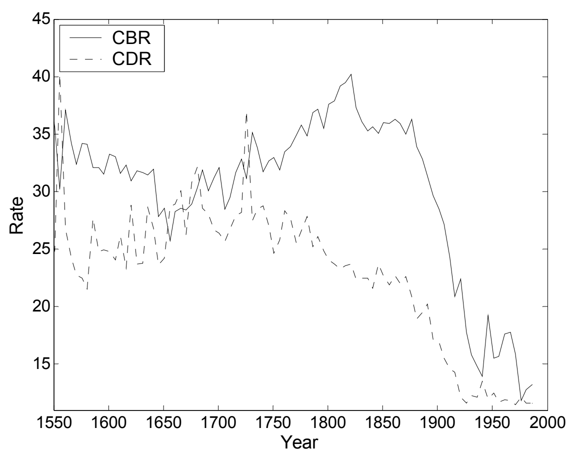

Figure 5. England: Demographic Transition

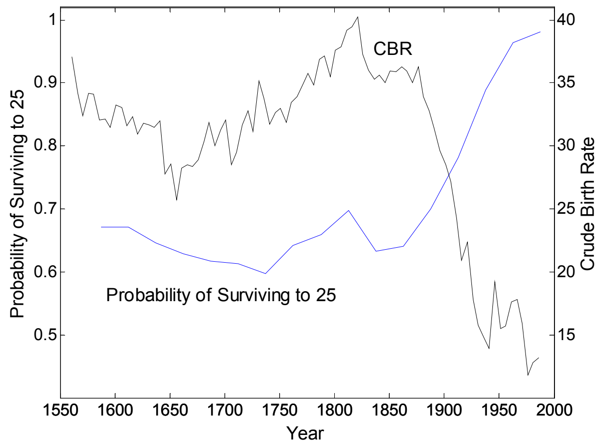

Figure 6. CBR and Young-Age Mortality

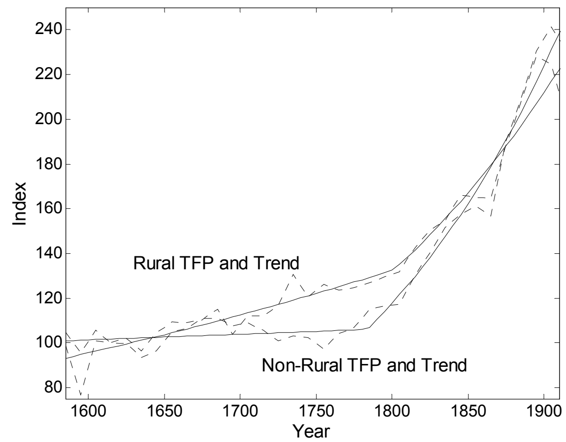

Figure 7. Estimated TFP

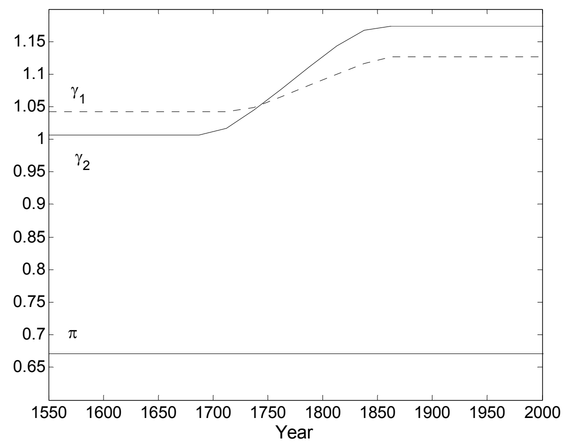

Figure 8. Experiment 1: Changing TFP

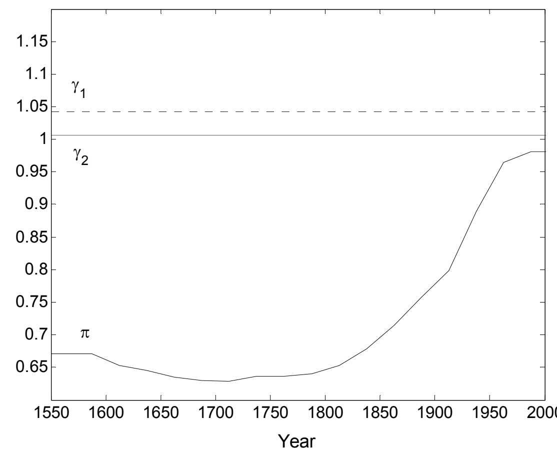

Figure 9. Experiment 2: Changing π

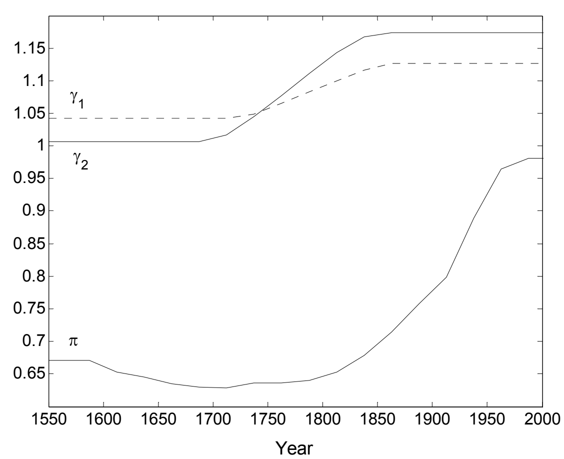

Figure 10. Experiment 3: Joint Experiment

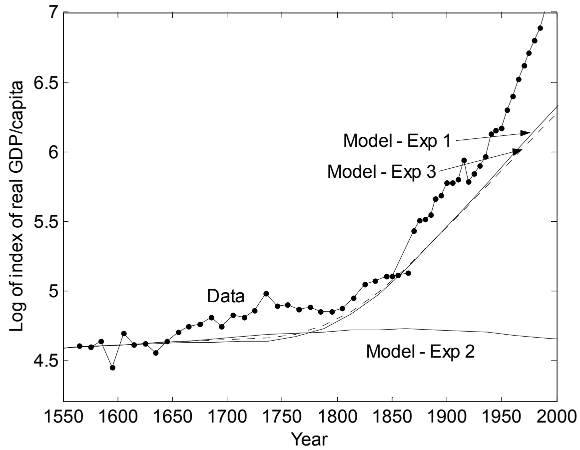

Figure 11. Model v. Data: Real GDP/capita

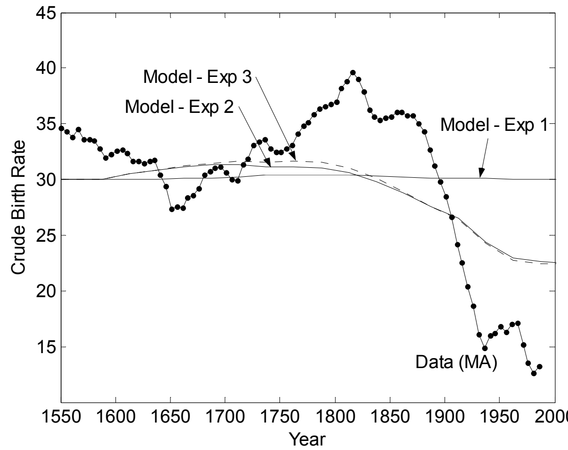

Figure 12. Model v. Data: CBR

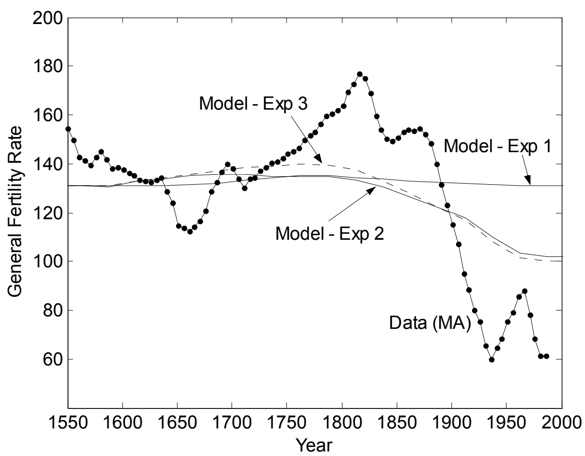

Figure 13. Model v. Data: GFR

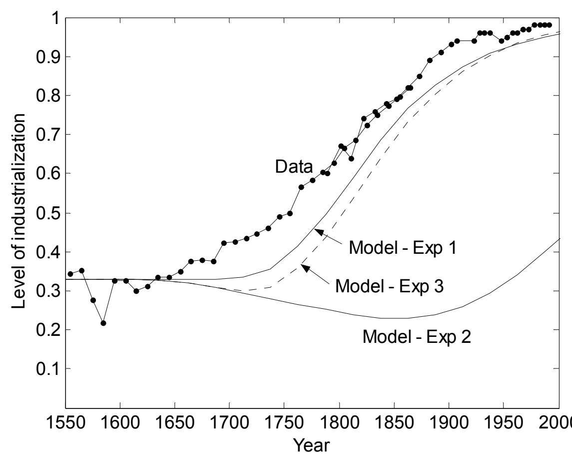

Figure 14. Model v. Data: Industrialization

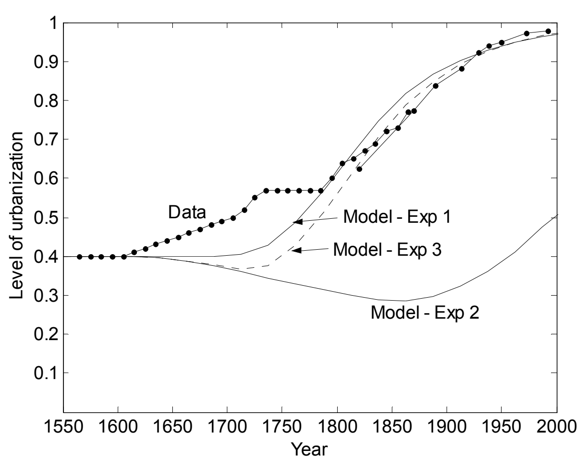

Figure 15. Model v. Data: Urbanization

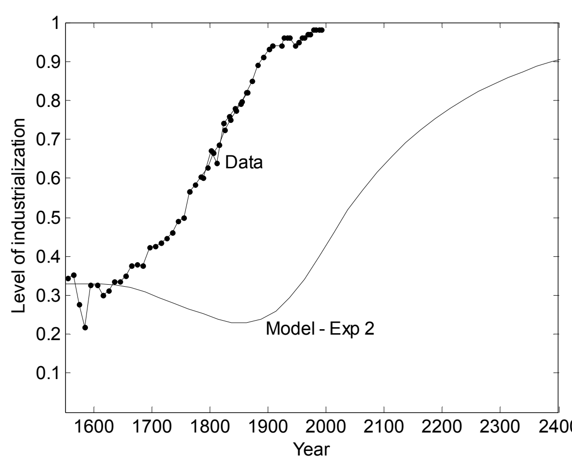

Figure 16. Exp 2: Industrialization

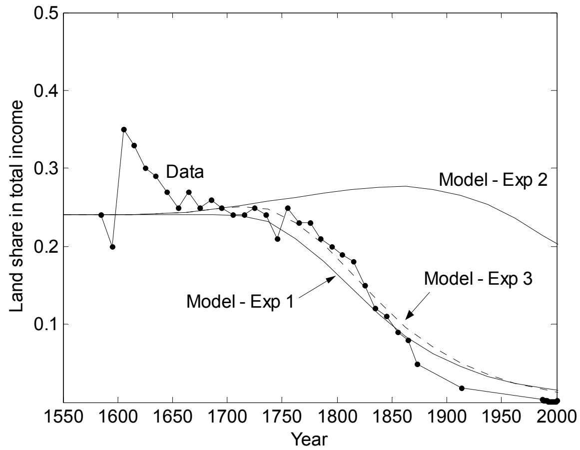

Figure 17. Model v. Data: Land Share

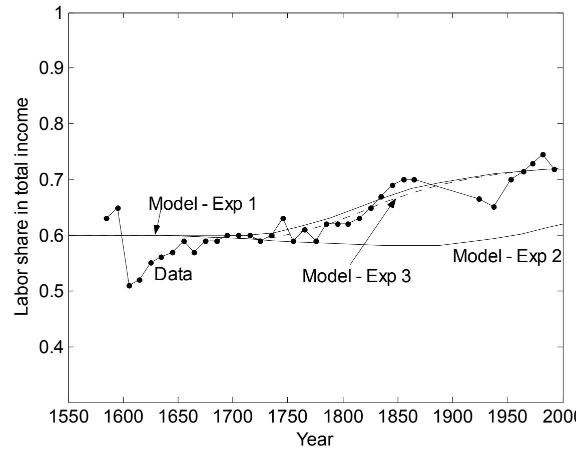

Figure 18. Model v. Data: Labor Share

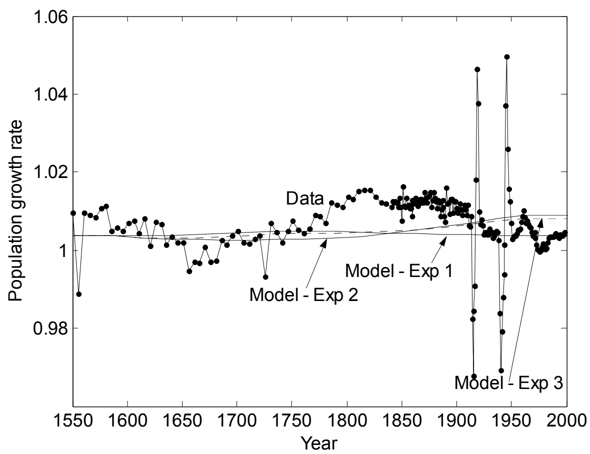

Figure 19. Model v. Data: Population Growth

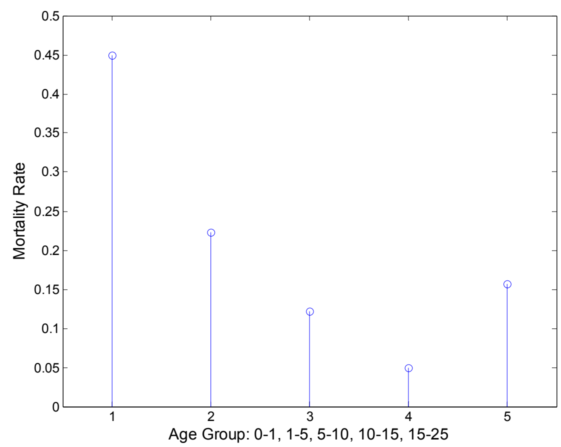

Figure 20. Child Mortality Distribution