Investor Information, Long-Run Risk, and the Duration of Risky Cash Flows

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Abstract

Value stocks have higher average returns than growth stocks. At the same time, the duration of value stocks' cash flows is considerably shorter than that of growth stocks. We show that when investors can fully distinguish short- and long-run consumption risk components of dividend growth innovations, only exposure to long-run consumption risk generates significant risk premia, implying that high-return value stocks should be long-duration assets, contrary to the historical data. By contrast, when investors observe the change in consumption and dividends each period but not the individual components of that change (limited information), exposure to short-run risk can generate large risk premia, implying that value stocks become short-duration assets while growth stocks are long-duration assets, as in the data. The limited information specifications we explore are not only consistent with the cash flow duration properties of value and growth stocks, they also explain the observed value premium, the higher Sharpe ratios of value stocks, the failure of the CAPM to account for the value premium, and the success of the HML factor of Fama and French (1993) in explaining the value premium.

JEL: G10, G12

1 Introduction

Empirical evidence shows that assets with low ratios of price to measures of fundamental value (value stocks) have higher average returns than assets with high ratios of price to fundamental value (growth stocks) (Graham and Dodd (1934); Fama and French (1992)). One explanation is that assets with high average returns command a high risk premium because they are more exposed to long-run cash flow risk. A leading example of this line of thought is presented by Bansal and Yaron (2004), who show that a small but extremely persistent common component in the time-series processes of consumption and dividend growth is capable of generating large risk premia and high Sharpe ratios simultaneously with a low and stable risk-free rate. A growing body of theoretical and empirical work is devoted to studying the role of long-run risk in consumption and dividend growth for explaining asset pricing behavior.[1] This line of thought suggests that value stocks must be more exposed to long-run cash flow risk than are growth stocks.

At the same time, a second strand of empirical evidence suggests that cash flow duration of value stocks is considerably shorter than that of growth stocks (Cornell (1999, 2000); Dechow, Sloan and Soliman (2004); Da (2005)). Shorter duration means that the timing of value stocks' cash flow fluctuations is weighted more toward the near future than toward the far future, whereas the opposite is true for growth stocks. Thus the duration perspective of equity seems to suggest that value stocks are less exposed to long-run cash flow risk than are growth stocks.

Can these seemingly contradictory findings be reconciled? In this paper we consider one possible reconciliation based on investor information about long-run risk. A maintained assumption in the theoretical models that study the role of small persistent long-run risk components of cash-flows is that investors can fully observe this component and distinguish its innovations from transitory shocks to consumption and dividend growth. We refer to this assumption as the *full information* specification. While this is a natural starting place and an important case to understand, in this paper we consider an alternative *limited information* specification in which market participants are faced with a signal extraction problem: they can observe the change in consumption and dividends each period, but they cannot observe

[1] See Parker (2001); Parker and Julliard (2004); Bansal, Dittmar and Kiku (2005); Bansal, Dittmar and Lundblad (2006) Hansen, Heaton and Li (2005); Kiku (2005); Malloy, Moskowitz and Vissing-Jorgensen (2005).

the individual components of that change.

A motivation for the limited information specification is that it is difficult or impossible to distinguish statistically between a purely i.i.d. process and one that incorporates a very small persistent component. Hansen et al. (2005), for example, find that the long-run riskiness of cash flows is hard to measure econometrically, and argue that such statistical challenges are likely to plague market participants as well as econometricians. Moreover, for some plausible specifications of the dividend process, the distinct roles of persistent and transitory shocks cannot be separately identified econometrically from the history of consumption and dividend data, even with an infinite amount of data. Thus, the full information assumption takes the amount of information investors have very seriously: market participants must not only understand that a small predictable component in cash-flow growth exists, they must also be able to decompose each period's innovation into its component sources, and have complete knowledge of how the shocks to these sources covary with one another, as well as knowledge of their relative importance in overall cash flow volatility.

We consider a model in which the dividend growth rates of individual assets are differentially exposed to two systematic risk components driven by aggregate consumption growth, in addition to a purely idiosyncratic component uncorrelated with aggregate consumption: one is a small but highly persistent (long-run) component as in Bansal and Yaron (2004), while the second is a transitory (short-run) i.i.d. component with much larger variance. In addition, by relying on the recursive utility specification developed by Epstein and Zin (1989, 1991) and Weil (1989), we presume that investors have preferences for which the intertemporal composition of risk matters, so that the relative exposure to short- versus long-run risks has a non-trivial influence on risk premia.

In this setting, there is more than one way to model the duration of individual assets' cash flows. A long duration asset may be modeled as one with cash flows that are highly exposed to the long-run risk component but are little exposed the short-run risk component, and vice versa for a short duration asset. Alternatively, the duration properties of individual assets may be modeled by recognizing that an equity claim is a portfolio of zero-coupon dividend claims with different maturities. It follows that growth firms, which are long duration assets, can be modeled as equity with relatively more weight on long-horizon zero-coupon dividend claims than value firms, which are short duration assets. We take both approaches to modeling duration in this paper.

We find that, when investors can fully distinguish the short- and long-run components of dividend growth innovations, assets that have high risk premia (value stocks) will be long-duration assets while those with low risk premia (growth stocks) are short-duration assets, contrary to the historical data. By contrast, under limited information, short-duration value assets can earn high risk premia while long-duration growth assets earn low risk premia, in line with the data. We show that plausible specifications under limited information can reproduce the magnitude of the spread in risk premia between value and growth stocks observed in the data, while at the same time preserving the key empirical implication that long-horizon equity is less risky than short-horizon equity. These results imply that limited information can be an important source of additional risk, and it can completely reverse the type of asset that commands high risk premia.

The intuition for this result is straightforward. When investors can observe the long-run component in cash flows–in which a small shock today can have a large impact on long-run growth rates–the long-run is correctly inferred to be more risky than the short-run, implying that long-duration assets must in equilibrium command high risk premia. Under limited information, when investors must perform a signal extraction problem, the opposite can occur: assets with high exposure to short-run consumption shocks command high risk premia because investors' optimal forecasts of the long-run component assign some weight to the possibility that such shocks will be persistent. At the same time, assets with low exposure to short-run consumption shocks command small risk premia because those assets appear to be largely dominated by the large idiosyncratic cash flow fluctuations that carry no risk premium.

An implication of these results is that, under full information, substantial cross-sectional variation in risk premia can only be generated by heterogeneity in the exposure to long-run consumption risk. By comparison, under limited information, substantial cross-sectional variation in risk premia can be generated by heterogeneity in the exposure to short-run, even i.i.d., consumption risk. In either case, however, the presence of long-run risk is central to delivering high risk premia, consistent with the insights of Bansal and Yaron (2004) and Hansen et al. (2005). The difference is that limited information generates a richer set of results, in which the relative exposure of cash flows to shocks with different degrees of persistence, and investors' perceptions of these shocks as seen through an optimal filtering lens, matters as much for risk premia as an asset's exposure to long-run consumption risk.

These results show that long-run consumption risk can be an important determinant of average returns even for short duration assets.

The limited information specifications we explore are not only consistent with the cash flow duration properties of value and growth stocks and the observed value premium, they also explain the higher empirical Sharpe ratios of value stocks and the failure of the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) to account for the value premium. In particular, the limited information specifications explain the high CAPM alphas of value stocks relative to growth stocks and the finding that there is little variation in the CAPM betas of growth stocks relative to value stocks (Fama and French (1992)). In addition, the limited information model is consistent with the ability of high-minus-low factor (HML) of Fama and French (1993) to explain the value premium.

Reconciling the cross-sectional properties of equity returns simultaneously with the cash flow duration properties of value and growth assets has proved a challenge for theoretical asset pricing. Lettau and Wachter (2006) use techniques from the affine term structure literature to develop a dynamic risk-based model that captures the value premium, the cash flow duration properties of value and growth portfolios, and the poor performance of the CAPM. However, Lettau and Wachter forgo modeling preferences and instead directly specify the stochastic discount factor. An essential element of their results is that the pricing kernel must contain additional state variables that can be at most weakly correlated with aggregate fundamentals. (Lettau and Wachter set this correlation to zero in their benchmark model.) By contrast, models that specify preferences directly as a function of aggregate fundamentals often have difficulty matching the cross-sectional properties of stock returns. For example, the habit model of Campbell and Cochrane (1999) has received significant attention for its ability to explain the time-series properties of aggregate stock market returns. But Lettau and Wachter (2006) and Wachter (2006) show that the Campbell and Cochrane model implies that assets with greater risk premia are long-horizon assets, rather than short-horizon assets, as in the data for value and growth portfolios. The full information specifications we explore here share this property with the Campbell and Cochrane model by counterfactually implying that assets with high risk premia are long duration assets. Santos and Veronesi (2005) modify the the Campbell and Cochrane model by adding cash flow risk for multiple risky assets and successfully generate a value premium for short-horizon assets. However, they also find that the cross-sectional dispersion in cash flow risk required to explain the magnitude of the premium is implausibly high. Other researchers have studied the cross-sectional properties of stock returns in production-based asset pricing models. Zhang (2005) shows that, when adjustment costs are asymmetric and the price of risk varies over time, growth assets can be less risky than assets in place (value stocks), consistent with the cash flow and return properties of value and growth assets. But the Zhang model does not account for the finding of Fama and French (1992) that value stocks do not have higher CAPM betas than growth stocks. In this paper we show that the combination of long-run risk and limited information is capable of reconciling the cross-sectional properties of value and growth assets with their quite different cash flow duration properties, all within a model of standard preferences driven by aggregate fundamentals.

The rest of this paper is organized as follows. The next section presents the asset pricing model and the model for cash flows. Section 3 presents theoretical results under the assumption that innovation variances in the cash flow model are constant, and shows how the signal extraction problem without learning influences equilibrium asset returns. Section 4 (to be completed) presents results from augmenting this model to include changing variances and learning. Section 5 concludes.

2 The Asset Pricing Model

Consider a representative agent who maximizes utility defined over aggregate consumption. To model utility, we use the more flexible version of the power utility model developed by Epstein and Zin (1989, 1991) and Weil (1989), also employed by other researchers who study the importance of long-run risks in cash flows (Bansal and Yaron (2004), Hansen et al. (2005) and Malloy et al. (2005)).

Let C_t denote consumption and $R_{C,t}$ denote the simple gross return on the portfolio of all invested wealth, which pays C_t as its dividend. The Epstein-Zin-Weil objective function is defined recursively as:

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}$$

where γ is the coefficient of risk aversion and the composite parameter $\theta = \frac{1-\gamma}{1-1/\Psi}$ implicitly defines the intertemporal elasticity of substitution Ψ.

Let $P_{j,t}^D$ denote the ex-dividend price of a claim to an asset that pays a dividend stream $D_{j,t}$ measured at the end of time t, and let P_t^C denote the ex-dividend price of a share of a claim to the aggregate consumption stream. From the first-order condition for optimal consumption choice and the definition of returns

$$E_t\left[M_{t+1}R_{C,t+1}\right] = 1, \qquad R_{C,t+1} = \frac{P_{t+1}^{C} + C_{t+1}}{P_t^D} \tag{1}$$

$$E_t\left[M_{t+1}R_{j,t+1}\right] = 1, \qquad R_{j,t+1} = \frac{P_{j,t+1}^{D} + D_{j,t+1}}{P_{j,t}^D} \tag{2}$$

where M_{t+1} is the stochastic discount factor, given under Epstein-Zin-Weil utility as

$$M_{t+1} = \left(\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right)^{\theta} R_{C,t+1}^{\theta-1}. \tag{3}$$

The return on a one-period risk-free asset whose value is known with certainty at time t is given by

$$R_{t+1}^{f} \equiv \left(E_t\left[M_{t+1}\right]\right)^{-1}.$$

2.1 A Cash Flow Model With Constant Variances

To study the role of informational assumptions in determining asset pricing behavior, we must first specify the stochastic processes for consumption and dividend growth rates. In what follows, we first describe the general form of the stochastic process for dividend growth and then say later how this form can be adapted to model individual asset's cash flows. We use lower case letters to denote log variables, e.g., $\log(C_t) \equiv c_t$.

The riskiness of any tradable asset in this economy is determined by the covariance its cash flows with the systematic risk factor M_{t+1}, where the latter depends directly on one-period-ahead consumption growth as well as indirectly on expected future consumption growth through the return to aggregate wealth, $R_{C,t+1}$. Thus we seek a model for cash flows that allows dividend growth rates to be potentially exposed to both transitory and persistent fluctuations in consumption. To model the persistent fluctuations, we follow Bansal and Yaron (2004) and assume that consumption and dividend growth rates contain a small predictable component x_t, which determines the conditional expectation of consumption growth:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma\varepsilon_{c,t+1} \tag{4}$$

$$\Delta d_{t+1} = \mu_d + \phi_x \underbrace{x_t}_{\text{LR risk}} + \phi_c \underbrace{\sigma\varepsilon_{c,t+1}}_{\text{SR risk}} + \sigma_{\varepsilon_d}\sigma\varepsilon_{d,t+1} \tag{5}$$

$$x_t = \rho x_{t-1} + \sigma_{\varepsilon_x}\sigma\varepsilon_{x,t} \tag{6}$$

$$\varepsilon_{c,t+1}, \varepsilon_{d,t+1}, \varepsilon_{x,t} \sim N.i.i.d\,(0,1)$$

The dividend specification (5) is closely related to a number of existing specifications studied in the literature. In particular, when $\phi_c = 0$ this specification is the same as that in Bansal and Yaron (2004). The term labeled "LR risk" captures the small long-run risk component emphasized in the literature because even very small innovations to x_t, if observable, can have large affects on valuation ratios and risk premia, as long as they are sufficiently persistent. In this paper we also allow dividend growth to be exposed to transitory consumption shocks, by introducing the additional component $\sigma\varepsilon_{c,t+1}$ with loading ϕ_c. We refer to this component as a short-run risk component, denoted above "SR risk," since its correlation with consumption growth and therefore the stochastic discount factor contributes to the riskiness of cash flows, but its purely transitory (i.i.d.) nature makes that risk short-lived. The loadings ϕ_x and ϕ_c govern the exposure of dividend growth to long-run and short-run consumption risk, respectively. Because the innovation $\varepsilon_{d,t+1}$ is uncorrelated with consumption growth, it does not contribute to the systematic risk of cash flows.

Under the limited information specification, investors recognize that there are separate long-run and short-run components to consumption and dividend growth in (4)-(6) but do not distinguish them, instead observing only the change in consumption and dividends each period. We assume that they form an estimate of the unobservable conditional means, x_t, and $x_{d,t} \equiv \phi_x x_t$, and that they do so optimally by sequentially updating a linear projection on the basis of data observed through date t.

Let $\widehat{x}_t$ and $\widehat{x}_{d,t}$ denote these optimal forecasts:

$$\begin{aligned}
\widehat{x}_t &\equiv \widehat{E}\left(x_t|\mathbf{z}^t\right) \\
\widehat{x}_{d,t} &\equiv \widehat{E}\left(x_{d,t}|\mathbf{z}_d^t\right)
\end{aligned}$$

where $\mathbf{z}^t$ and $\mathbf{z}_d^t$ are vectors containing the history of consumption and dividend data, respectively, through time t. It is straightforward to express the dynamic system (4)-(6) in state space representations for consumption and dividend growth and use the Kalman filter to calculate the estimates $\widehat{x}_t$ and $\widehat{x}_{d,t}$ recursively. In doing so, we use the steady-state Kalman filter, effectively assuming that agents have an infinite amount of data from which to base their forecasts. Thus, under limited information, investors observe not the system (4)-(6) that generates the data, but instead an innovations representation based on the optimal estimates $\widehat{x}_t$ and $\widehat{x}_{d,t}$:

$$\begin{aligned}
\Delta c_{t+1} &= \widehat{x}_t + v_{c,t+1} && (7)\\
\widehat{x}_{t+1} &= \rho\widehat{x}_t + K v_{c,t+1} && (8)\\
\Delta d_{t+1} &= \widehat{x}_{d,t} + v_{d,t+1} && (9)\\
\widehat{x}_{d,t+1} &= \rho\widehat{x}_{d,t} + K^d v_{d,t+1}, && (10)
\end{aligned}$$

where K and K^d are the steady state Kalman gain parameters associated with the state space representations for consumption and dividend growth, respectively. The innovations $v_{c,t+1}$ and $v_{d,t+1}$ will in general be correlated, and are composites of the underlying innovations in (4)-(6).[2]

The state space representation provides a convenient way to calculate the likelihood function for the consumption and dividend processes given in (4)-(6). In the absence of apriori restrictions on the state space parameters, however, those parameters are not identified from the history of consumption and dividend data, even with an infinite amount of data. In fact, the likelihood functions for the innovations representations of Δc_{t+1} and Δd_{t+1} in (7)-(10) are the same as those implied by the system (4)-(6). Consequently, an econometrician armed with observations on consumption and dividends would be unable to observe x_t or to separately identify the parameters in (4)-(6). A modeling implication of this observation is that calibration exercises which assume that agents can observe the persistent component x_t also implicitly assume that market participants have more information than do econometricians with historical data on consumption and dividends. In practice, theorists use information on risk premia to calibrate the parameters in (4)-(6), implying that the model's predictions for asset prices are no longer determined from purely exogenous driving processes for consumption and dividends.

For the full information specification, x_t summarizes the information upon which conditional expectations are based. Solutions to the model's equilibrium price-consumption and

[2] The system (7)-(10) can also be expressed as a pair of ARMA(1,1) processes for Δc_{t+1} and Δd_{t+1}:

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c (1-\rho) + \rho\Delta c_t + v_{c,t+1} - b_c v_{c,t} && (11)\\
\Delta d_{t+1} &= \mu_d (1-\rho) + \rho\Delta d_t + v_{d,t+1} - b_d v_{d,t}, && (12)
\end{aligned}$$

where the parameters $b_c \equiv (\rho - K)$, $b_d \equiv \left(\rho - K^d\right)$ and the variance-covariance matrix of $v_{c,t+1}$ and $v_{d,t+1}$ are functions of ϕ_x, ϕ_c, and variance-covariance matrix for the fundamental shocks $\varepsilon_{c,t+1}$, $\varepsilon_{x,t+1}$ and $\varepsilon_{d,t+1}$.

price-dividend ratios are found by iterating on the Euler equations (1) and (2), assuming that individuals observe the consumption and dividend processes in (4) and (5). This delivers a policy function for the price-consumption and price-dividend ratios as a function of a single state variable x_t.

In the limited information specification, equilibrium price-consumption and price-dividend ratios are calculated assuming individuals only observe the composite shock processes given in (7)-(10), even though shocks to the consumption and dividend processes are actually generated by the individual shocks in (4)-(6). In this case, the policy function for the price-consumption ratio is a function of $\widehat{x}_t$, while the price-dividend ratio is a function of both $\widehat{x}_t$ and $\widehat{x}_{d,t}$. For each case, we simulate histories for consumption and dividend growth based on the processes in (4)-(6) and use solutions to the policy functions to generate equilibrium paths for asset prices. The process is iterated forward to obtain simulated histories for asset returns.[3] The Appendix explains how we solve for these functional equations numerically on a grid of values for the state variables.

3 Theoretical Results

To investigate the role of investor information in influencing risk premia, we begin by investigating the model's implications for summary statistics on the price-dividend ratio, excess returns, and risk-free rate under limited as compared to full information. Tables 1 and 2 present results of this form. The output is generated by simulating 1000 samples of size 840 months, computing annual returns from monthly data, and reporting the average statistics for annual returns across the 1000 simulations. To make our results comparable to the existing literature on long-run risk, the results in Table 1 are based on parameters set at monthly frequency as in Bansal and Yaron (2004) as follows: $\delta = 0.998985$, $\mu_d = \mu_d = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\sigma_{\varepsilon_x} = 0.044$, $\Psi = 1.5$, $\gamma = 10$. Notice that the innovation variance in x_t is small relative to the overall volatility of consumption: the standard deviation of ε_x is only 4.4% of the standard deviation of the consumption growth innovation ε. Notice also

[3] The one minor complication in the simulations is that the policy functions for the limited information specifications are a function of the current innovation in the composite processes that appear in ((7)-(10), whereas the actual innovations are generated from (4)-(6). However, the moving average representations of (7) and (9) are invertible, and the innovations $v_{c,t}$ and $v_{d,t}$ can be recovered from the sums $\sum_i b_c^i (\Delta c_{t-i} - \rho \Delta c_{t-i-1} - \mu_c)$ and $\sum_i b_d^i (\Delta d_{t-i} - \rho \Delta d_{t-i-1} - \mu_d)$, respectively.

that the persistence of x_t is set to be high; as Bansal and Yaron point out, this is important for generating significant risk premia from small innovations to cash flows. We set σ_{ε_d} equal to 6 rather than 4.5, slightly higher than in Bansal and Yaron, in order to better match the correlation in our model between consumption and dividends observed in the data. The table considers a range of values for the parameters ϕ_c and ϕ_x, which govern the exposure to short-run and long-run consumption risk, respectively . In what follows, we denote the log return on the dividend claim $r_{j,t+1} = \ln(R_{j,t+1})$ and the log return on the risk-free rate $r_{f,t+1} \equiv \ln\left(R^f_{t+1}\right)$.

Table 1 shows that the impact on risk premia of exposure to long-run risk versus short-run consumption risk is sensitive to assumptions made about investor information. The existing literature based on full information generates a high risk premium by placing significant weight ϕ_x on the long-run risk component with small variance, while at the same time assigning little or no role for short-run risk. For example, Bansal and Yaron (2004) set $\phi_x = 3$ and $\phi_c = 0$. With this in mind, several related aspects of Table 1 are worthy of emphasis.

First, when exposure to short-run risk is sufficiently large, the limited information specification generates a substantially higher risk premium than the full information specification (e.g., row 2 of Table 1). Second, the limited information specification generates a small risk premium whenever exposure to short-run risk is small. For example, even when exposure to long-run risk, ϕ_x, is as high as 3, the log risk premium $E(r_i - r_f)$ is only 1.25% per annum if ϕ_c is as small as 2.2 (row 1). In fact, under the parameterization specified above, values for ϕ_c that are much smaller than 2.2 are ruled out in the limited information case by the requirement that the price-dividend ratio be finite.[4] This is analogous to the requirement in the Gordon growth model that the expected stock return be greater than the expected dividend growth rate to keep the price-dividend ratio finite.

Third, under full information, substantial cross-sectional variation in risk premia can only be generated by heterogeneity in the exposure to long-run consumption risk. For example, when $\phi_x = 3$ and ϕ_c is increased from 2.2 to 6, the log risk premium $E(r_i - r_f)$ increases by just one and a quarter percent, from 5.15% to 6.40% per annum (compare rows 5 and 6 of Table 1). By comparison, under limited information, substantial cross-sectional variation

[4] Fixing ϕ_x, as $\phi_c \to 0$, the risk premium in the limited information case converges to a very small number, differing from zero only by a Jensen's inequality term.

in risk premia can be generated by heterogeneity in the exposure to short-run consumption risk: when $\phi_x = 3$ and ϕ_c is increased from 2.2 to 6, the log risk premium increases by almost 8 percentage points from 1.26% to 8.42% per annum. On the other hand, fixing ϕ_c and varying ϕ_x generates little variation in risk premia under limited information.

To understand these results, recall that the risk premium on any asset is determined by the covariance between M_t and the innovation in the equity return. The innovation in the equity return can be decomposed into a component based on revisions in expectations (news) of future dividend growth and a component based on revisions in expectations about future returns. Revisions in expectations about future returns are relatively unimportant in the present version of the model because we have not introduced mechanisms such as changing consumption and dividend volatility for generating time-varying risk premia on the asset. Thus risk premia here are largely determined by covariance between M_t and news about future cash-flow growth.

With risk premia determined by the covariance between M_t and news about future cash-flow growth, there are two offsetting effects on the equity premium for the full information case as compared to the limited information case. First, when an innovation ε_x to the persistent component of consumption and dividend growth occurs, the solution to the optimal filtering problem implies that investors with limited information assign some weight to the possibility that the shock is transitory (coming from ε_c or ε_d) and will therefore fail to revise their expectation of future dividend growth as much as they would under full information. This contributes to a greater risk premium in the full information case as compared to the limited information case. Second, when an innovation ε_c to the short-run risk component occurs, the solution to the optimal filtering problem implies that investors with limited information will assign some weight to the possibility that the shock is persistent (coming from the long-run risk component), and will therefore revise their expectation of future dividend growth more than they would under full information. This contributes to a greater risk premium in the limited information case as compared to the full information case.

Notice that when ϕ_x is large (e.g., equal to 3 Table 1) and ϕ_c relatively small (e.g., 2.2), the risk premium in the full information case can be substantial while the premium in the limited information case is quite small. For such parameter values, the first effect dominates the second. Intuitively, this occurs because when exposure ϕ_c to short-run risk is small and the long-run risk component has small variance, the innovations $\varepsilon_{c,t+1}$ and $\varepsilon_{x,t+1}$ receive little weight in the composite consumption and dividend shocks $v_{c,t+1}$ and $v_{d,t+1}$ generated from the Kalman filter. Instead, these innovations are largely dominated by the more volatile idiosyncratic cash flow shocks $\varepsilon_{d,t+1}$ that carry no risk premium. This explains why these cases generate a low risk premium under limited information.

By contrast, when when ϕ_x is small (e.g., equal to 1 Table 1) and ϕ_c relatively large (e.g., 6), the risk premium in the limited information case can be substantial while the premium in the full information case is quite small. For these parameter values, the second effect dominates the first. Here, the *i.i.d.* innovation $\varepsilon_{c,t+1}$, which under limited information cannot be distinguished from the persistent $\varepsilon_{x,t+1}$ shock, receives significant weight in the composite shocks $v_{c,t+1}$ and $v_{d,t+1}$ and therefore generates significant revisions in expectations of future dividend growth long into the future. Moreover, since $\varepsilon_{c,t+1}$ affects consumption growth, it generates non-negligible correlation between $v_{c,t+1}$ and $v_{d,t+1}$, and therefore between M_t and innovations to $r_{j,t}$. The result is a higher risk premium under limited information than under full information.

The spread in log risk premia that may be obtained by varying ϕ_x and ϕ_c can be made larger by altering parameter values. Table 2 shows the same statistics as those in Table 1 when γ is 15 instead of 10, Ψ is 1.3 instead of 1.5, σ_{ε_x} is 0.10 instead of 0.044, and δ is 0.993 instead of 0.98985. The other parameters are the same as those used to produce the results in Table 1. In this case the variance of the persistent component x_t has been made slightly larger, though still substantially smaller than the innovation variance for consumption growth. As a result, smaller values of the loadings ϕ_x and ϕ_c are required to generate large risk premia. Other than this, the main features of Table 1 are preserved in Table 2, but larger risk premia and a greater spread in risk premia are obtained.

Note that the reported price-dividend ratios in the table, which are generally lower than those in the data, are not readily comparable to their empirical counterparts for actual firms. This is because the hypothetical firms in the model, with cash flow process of the form (4)-(6), have no debt and do not retain earnings. Thus, the dividends in the model are more analogous to free cash flow than to actual dividends, implying that price-dividend ratios in the model should be lower than measured price-dividend ratios in historical data.

3.1 Heterogeneity In Consumption Risk Exposure

How can these results be used to model the cash flow and return properties of value and growth stocks? Our goal is to reconcile the cross-sectional properties of returns with the cash flow duration properties of value and growth assets. The results above suggest that one way we may accomplish this is by modeling firms as having varying degrees of exposure to short- and long-run consumption risk. In this setting, a long duration (growth) asset may be modeled as one with cash flows that are highly exposed to the long-run consumption risk component, with high ϕ_x, but are little exposed the short-run risk component, having low ϕ_c, and vice versa for a short duration (value) asset. When ϕ_x is small and ϕ_c is large, the timing of cash flow fluctuations is weighted more toward the near future than the far, implying the asset's cash flows are of shorter duration than assets for which the loading ϕ_x is large and ϕ_c is small.

Under limited information, it is short duration assets, those with relatively low exposure to long-run consumption risk and high exposure to short-run consumption risk (e.g., row 2 of Table 2), that have high risk premia and low price-dividend ratios, consistent with the properties of value stocks in the data. At the same time, it is long duration assets, those with high ϕ_x and low ϕ_c (e.g., row 6 of Table 2), that have lower risk premia and higher price-dividend ratios, consistent with the properties of growth stocks in the data.

The results are quite the opposite under full information. Short duration assets, those with relatively low exposure to long-run consumption risk and high exposure to short-run consumption risk (e.g., row 2 of Table 2), have low risk premia and high price-dividend ratios, whereas long duration assets, those with high ϕ_x and low ϕ_c (e.g., row 6 of Table 2), have high risk premia and low price-dividend ratios.

These findings are illustrated graphically in Figure 1, which plots annualized price-dividend ratios as a function of the ratio of long-run to short-run consumption risk exposure, ϕ_x/ϕ_c. For this figure, the ratio ϕ_x/ϕ_c is varied in such as way as to hold fixed the 15-month variance of dividend growth that is attributable to the consumption innovations. The two left panels plot the steady-state price dividend ratio under limited information. The the left-most panel plots this ratio at the steady state value of $\widehat{x}_t$, along with plus and minus two standard deviations around steady state in $\widehat{x}_t$ (holding fixed $\widehat{x}_{d,t}$ at its steady-state level). The middle panel plots plots the price-dividend ratio at the steady state value of $\widehat{x}_{d,t}$, along with plus and minus two standard deviations around steady state in $\widehat{x}_{d,t}$ (holding fixed $\widehat{x}_t$ at its steady-state level). The right-most panel plots the price-dividend ratio under full information as a function of ϕ_x/ϕ_c, plus and minus two standard deviations around steady state in x_t.

The plots are upward sloping under limited information but downward sloping under full information. Recall that price-dividend ratios are high when risk premia are low, and vice versa. This shows that assets with cash flows that load heavily on the long-run component, x_t, are *more* risky under full information but *less* risky under limited information. Thus, under full information, assets more exposed to long-run cash flow fluctuations carry higher risk premia, while under limited information, assets more exposed to short-run cash flow fluctuations carry high risk premia.

We close this section by briefly making one observation about the cash flow betas studied in Bansal et al. (2006). Bansal et. al. point out that regressions of dividend growth on 4 and 8 quarter trailing moving averages of consumption growth, where the slope coefficient in this regression is called the 'cash flow beta,' show that value stocks have higher cash flow betas than growth stocks.[5] It is clear that heterogeneity in ϕ_x, governing exposure to long-run consumption risk, can generate heterogeneity in cash flow betas with respect to moving averages of consumption growth over longer horizons. A brief section in the Appendix shows that—when consumption and dividend data are time-aggregated, as in the historical data—heterogeneity in ϕ_c, governing exposure to i.i.d. consumption risk, can also generate heterogeneity in cash flow betas with respect to moving averages of consumption growth over 4 or 8 quarter horizons.

In the next section we consider an alternative way of modeling individual assets with different cash flow properties, by specifying firms as having different time-varying weights in economy-wide dividend payouts at different maturities.

3.2 Modeling firms: Zero-Coupon Dividend Claims

An equity claim is a portfolio of zero-coupon dividend claims with different maturities. It follows that one way to model firms with different cash flow duration properties is to specify assets as having time-varying shares in a sequence of "market" dividend claims, $\{D_t\}_{t=0}^{\infty}$, with different maturities. Here firms differ only in the timing of their cash flows, allowing us

[5] One caveat with this observation is that the cash flow betas are measured with considerable error, and therefore are not statistically distinguishable from one another.

to isolate the role of cash flow duration in generating cross-sectional differences in expected returns. Long-duration growth firms are modeled as equity with relatively more weight placed on long-horizon dividend claims, while short-duration value firms are modeled as equity with relatively more weight placed on short-horizon dividend claims. The approach has been used in previous work by Lynch (2003), Menzly, Santos and Veronesi (2004), Santos and Veronesi (2005), and Lettau and Wachter (2006). Long-lived assets are modeled as having price-dividend ratios that together sum to the aggregate market price-dividend ratio.

The first step in this modeling strategy is to specify how zero-coupon equity is valued under limited and full information. Denote $P_{n,t}$ as the price of an asset that pays the aggregate dividend n periods from now, and $R_{n,t}$ the one-period return on zero-coupon equity maturing in n periods:

$$R_{n,t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}.$$

The zero-coupon equity claims are price under no-arbitrage according to the following Euler equation:

$$\begin{aligned} E_t \left[M_{t+1} R_{n,t+1} \right] &= 1 \Longrightarrow \\ P_{n,t} &= E_t \left[M_{t+1} P_{n-1,t+1} \right] \\ P_{0,t} &= D_t, \end{aligned}$$

where the process for cash flows that generates the data D_t is given by (4)-(6).

The appendix provides detailed information on how the recursion above is solved numerically. Denote $r_{n,t+1} = \ln \left(R_{n,t+1} \right)$. Note that, since the aggregate market is the claim to all future dividends, its price-dividend ratio is the sum of the ratios $\sum_{n=1}^{\infty} P_{n,t}/D_t$. Plotting $R_{n,t+1}$ against n produces a yield curve, or term structure, of zero-coupon dividend claims.

The time-varying share processes for each firm are modeled following Lettau and Wachter (2006). We outline only the main aspects of this approach here and refer the reader to that article for further detail. Consider a sequence of $i = 1, ..., N$ portfolios of firms at the same life-cycle stage, (hereafter referred to simply as 'firms' for brevity). The firms pay a share, $s_{i,t+1}$, of the aggregate dividend D_{t+1} at time $t+1$. Shares are greater than zero and sum to unity across all $i = 1, ..., N$. The share process is deterministic, with $\underline{s}$ being the lowest share of a firm in the economy. Firms experience a life-cycle in which this share grows deterministically at a rate g_s until reaching a peak $s_{i,N/2+1} = (1 + g_s)^{N/2} \underline{s}$ and then shrinks deterministically at rate g_s until reaching $s_{i,N+1} = \underline{s}$. The cycle then repeats itself. Firms are identical except that their life-cycles are out-of-phase, i.e.., firm 1 starts at $\underline{s}$, firm 2 at $(1+g_s)\,\underline{s}$, and so on. The parameter g_s is set to 1.67% per month, or 20% per year, as in Lettau and Wachter (2006).

Before discussing the properties of portfolios of firms sorted on the basis of price-dividend ratios, it is instructive to compare the term structure of equity under limited and full information. Figures 2 and 4 plot summary statistics for returns as a function of maturity, n, under two parameter configurations described in the notes to the figures. Similar figures are presented in Lettau and Wachter (2006) and Hansen et al. (2005) (discussed below), but for a different asset pricing model, M_{t+1}.

Under limited information, the annualized log risk premium declines with maturity. The largest spread from short- to long-maturities occurs under the parameterization of Figure 4: the log risk premium is 15% per annum for equity that pays a dividend one month from now and 5% per annum for equity that pays a dividend 15 years from now. Figure 2 displays similar results using the parameterization that generated the results in Table 2, but the spread between short- and long-horizon equity is a bit less. This spread is greater in Figure 4 by increasing risk aversion from 15 to 16 and reducing the intertemporal elasticity of substitution from 1.3 to 1.2. This suggests that the limited information specification has the potential to explain the higher mean excess returns of short-duration assets as compared to long-duration assets found in the data.

Under full information, the annualized log risk premium increases with maturity. The log risk premium is 1.8% per annum for equity that pays a dividend one month from now and 5.5% per annum for equity that pays a dividend 15 years from now. The long-run is more risky and, as such, long-duration assets carry high risk premia.

The middle panels of Figures 2 and 4 show that in both limited and full information, volatility increases with the horizon. But the bottom panels show that the Sharpe ratios decrease with the horizon under limited information while they rise with the horizon under full information. This suggests that the limited information specification, in contrast to the full information specification, has the potential to explain the higher Sharpe ratios of short-duration assets as compared to long-horizon assets.

Hansen et al. (2005) present zero-coupon equity plots for price-dividend ratios $P_{n,t}/D_t$ rather than mean excess returns as in Figures 2 and 4. Since high price-dividend ratios correspond to low excess returns, the two plots are essentially mirror-images of one another.

Their plots are based on the same Epstein-Zin-Weil model of preferences used here, but the results are formed from historical data and somewhat different parameter values. Below we interpret value and growth firms as having different, time-varying shares in the aggregate dividend, but it is also possible to interpret value and growth firms distinguished by heterogeneity in the loadings ϕ_c and ϕ_x, as in the previous subsection. Regardless of the values of ϕ_c and ϕ_x, results (not reported) indicate that, in the cash flow models we study, the term structure of equity is always downward sloping under limited information, while it is always upward sloping under full information.[6] Changing the loadings ϕ_c and ϕ_x merely changes the slope of the term structure, it does not change the sign of the slope. These findings differ somewhat from those of Hansen et al. (2005), who report that—when risk-aversion is sufficiently high—the price-dividend term-structures of value and growth portfolios have slopes of opposite signs. There are, however, a number of discrepancies between our analyses that could account for these differences. Hansen et al. (2005) use different preference parameter values, for example restricting Ψ to be within a neighborhood of unity, whereas most of our parameterizations use larger values for Ψ. They also use a different model of cash flows, in which consumption and corporate earnings are cointegrated and consumption growth follows a multivariate first-order Markov process.

Fama and French (1992) pointed out that the CAPM fails to explain the return premium on short-duration value stocks over long-duration growth stocks. To relate our findings to these results, Figures 3 and 5 plot the results of CAPM regressions of zero-coupon equity returns on the excess market return, as a function of maturity. The top panel shows the CAPM betas and the bottom panel shows the CAPM alphas. The results for limited information and full information are plotted simultaneously in the figures on separate scales (limited information on the left, full information on the right). As above, returns are are converted to percent per annum. The figures show that, under limited information, the shortest-duration equity have high alphas (as high as 8% in Figure 3 and 9% in Figure 5 for equity that pays a dividend in one month), whereas the longest-maturity equity have small alphas (in both figures close to -2% for equity that pays a dividend 15 years from now). This is reminiscent of the findings of Fama and French (1992), in which short-duration value assets display relatively large positive CAPM alphas, while long-duration growth as-

[6] This is true as long as parameter values are set so that greater exposure to x_t makes an asset riskier rather than providing insurance. In a long-run "insurance" model, the full information term structure slopes down, but overall risk premia are very low or even negative.

sets have smaller (in absolute value) negative alphas. Under full information, there is much less variation in the alphas with maturity and the variation goes the wrong way: alphas of short-duration assets are always lower than those of long-duration assets. The shortest-duration equity displays alphas of about -3% in both figures, while the longest-duration equity has alphas of about 0.3%. The bottom panels show that, under limited information, long-duration equity—despite its having lower expected excess returns than short-duration equity—has slightly higher CAPM betas, as in the data (Fama and French (1992)).

The full information specifications described above, with their upward sloping term structures of equity, make it difficult to explain why short-horizon assets are more risky than long-horizon assets. The reason is simple: when agents can perfectly observe x_t, the long-run appears quite risky, implying that assets which pay a dividend far into the future command a high risk premium. By contrast, the results above suggest that the limited information specification, with its downward sloping term structure of equity, has the potential to explain the value spread in a manner consistent with the quite different cash flow duration properties of value and growth assets. Under limited information, assets with more weight in low-maturity equity will be short-duration assets and simultaneously have higher expected returns and lower price-dividend ratios than long-duration assets with more weight in distant-maturity equity. Next we develop the quantitative implications of these features of limited information by focusing on the behavior of portfolios of firms.

Since each firm pays a dividend $s_{i,t+1}D_{t+1}$, $s_{i,t+2}D_{t+2}, \ldots$, no arbitrage implies that the ex-dividend price of firm i at time $t+1$ is given by

$$P_{i,t} = \sum_{n=1}^{\infty} s_{i,t+n} P_{n,t}.$$

When $s_{i,t+1}$ is low, dividend payments are low today but will be high in the future when n is large; these are long-duration assets with greater weight placed on distant-maturity dividend claims. We have already seen that, under limited information, those assets have low risk premia and high price-dividend ratios. When $s_{i,t+1}$ is high, dividend payments are high today but will be low in the future; these are short-duration assets with greater weight placed on short-maturity dividend claims. We have already seen that, under limited information, these assets have high risk premia and low price-dividend ratios. Thus, firms move through their life-cycle over time by starting as long-duration growth assets, placing most of their weight in long-maturity zero-coupon dividend claims, slowly shifting to short-duration value assets with most of their weight in short-maturity zero-coupon dividend claims.

To create portfolio returns, we simulate a time-series for dividends and prices and, using the share process described above, form portfolios of N firms, (where N is chosen to be some large number),[7] by sorting firms into deciles based on their price-dividend ratios and then forming equally-weighted portfolios of the firms in each decile. The portfolios are rebalanced every simulation year. The purpose of this procedure is to create portfolios of firms that display heterogeneity in the timing of their dividend payments, and thus heterogeneity in the duration of their cash flows.

Tables 3 and 4 present summary statistics for the decile portfolios under the same two parameter configurations used to generate the zero-coupon equity results in Figures 2 and 3, and 4 and 5, respectively. The statistics are presented for expected excess returns, Sharpe ratios, and CAPM regressions, based on a single long simulation of the data generating process in (4)-(6). We refer to the portfolio in the highest price-dividend decile as the growth portfolio, denoted G in the tables, and the portfolio in the lowest price-dividend decile as the value portfolio, denoted V in the tables. We present these results only for the limited information specifications, since, for the reasons described above, specifications with full information generate a value premium by counterfactually making long-duration assets more risky than short-duration assets.

Under the parameter configuration of Table 3, the mean excess return on the growth portfolio is 7.13%, while that of the value portfolio is 11.40%, leaving a spread between the two of 4.27% . These numbers are close to those found in the data. For example, Hansen et al. (2005) report that the mean excess return in the lowest book equity-to-market capitalization quintile (B/M quintile) has a annual return of 7.91%, while that in the highest B/M quintile has a return of 12.69%, implying a spread of 4.8%. Table 4 shows that it is straightforward to alter parameter values to come even closer to matching these statistics from the historical data: as above, by increasing risk aversion from 15 to 16 and reducing the intertemporal elasticity of substitution from 1.3 to 1.2. In this case, the mean excess return on the growth portfolio is 7.38%, while that of the value portfolio is 12.1%, leaving a spread between the two of 4.7%. The limited information specifications also predict that Sharpe ratios rise when moving from growth to value portfolios, as in the data. For example, in Table 4, the Sharpe ratio of the growth portfolio is 0.31, while that of the value portfolio is 0.52. In the post-war

[7] We set the number of firms to be 1020, implying a 1020 month, or 85 year life-cycle for a firm.

data, the lowest B/M quintile has a Sharpe ratio of 0.32 and the highest has a Sharpe ratio of 0.57 (Lettau and Wachter (2006)).

The second panel of Tables 4 and 5 display CAPM alphas and betas implied by simulations of the limited information models. Recall that the market portfolio in the model is given by the sum across all firms of the individual firm price-dividend ratios. Fama and French (1992) showed that value portfolios have positive CAPM alphas, while growth portfolios had negative alphas. In addition, value portfolios have slightly lower CAPM betas than growth portfolios. The same is true in the limited information specifications we investigate: Table 4, for example, shows that alphas rise from about -1.7% for the growth portfolio to 3.19% for the value portfolio. By comparison, in the post-war data, the lowest B/M quintile has an alpha of -1.7% and the highest 4.0% (Lettau and Wachter (2006)). Finally, the third panel of Tables 3 and 4 show the results of adding the HML (high-minus-low) factor of Fama and French (1993) as an additional regressor in CAPM time-series regressions of the excess portfolio returns onto the excess market return. HML is constructed as the return on a portfolio short in the extreme growth decile and long in the extreme value decile. Consistent with the classic empirical findings of Fama and French (1993), the model implies that adding HML as an additional factor drastically reduces the magnitude of the CAPM alphas in all decile portfolios.

4 Adding Time-Varying Uncertainty

To be completed.

5 Conclusion

An important recent strand of asset pricing literature has emphasized the potential role of long-run consumption risk for explaining salient asset pricing phenomena. Because any long-run component of consumption must necessarily represent a small fraction of short-run cash flow volatility, econometricians face concrete statistical hurdles in attempting to identify such components from data. The goal of this paper is to take one step toward understanding how equilibrium asset prices might be affected if market participants have the same difficulties as econometricians observing small long-run components in firm cash flows.

We find that if investors can only observe the history of consumption and dividend changes, but not the individual components of those changes, the asset pricing implications can be quite different from a full information world in which market participants fully discern the distinct roles of persistent and transitory shocks. Under some parameter configurations, such limited information causes market participants to demand a higher premium for engaging in risky assets than would be the case under full information. Alternatively, the same risk premium may be achieved with lower relative risk aversion under a model of limited information than for the same model under full information. This is of interest in its own right, as it is often argued that modern-day asset pricing models are unlikely to explain high risk premia without appealing to high risk aversion.[8]

More importantly, we find that the assumptions we make about investor information can have important implications for the cash flow duration perspective of value and growth assets. In particular, the models we study imply that, under limited information, the term structure of equity is downward sloping. Thus, value stocks can be made consistent with empirical evidence that the cash flow duration of these assets is considerably shorter than that of growth stocks. By contrast, when investors can fully distinguish the short- and long-run components of dividend growth innovations, the term structure of equity is upward sloping. Thus, value stocks must be long-duration assets, while growth stocks are short-duration assets, at odds with empirical evidence to the contrary. The reason is simply that when investors can perfectly observe the long-run component in cash flows, the long run appears very risky; thus assets exposed to dividend fluctuations far into the future carry high risk premia.

Under limited information, substantial dispersion in risk premia across assets may be generated by heterogeneity in the exposure to short-run consumption risk. As such, assets that have small exposure to long-run consumption risk but are highly exposed to short-run, even i.i.d., consumption risk can command high risk premia. This is not the case under the full information models we study. These findings do not, however, diminish the importance of long-run risk in generating high risk premia. Indeed, without both long- and short-run consumption risk, there is no signal extraction problem and no way for heterogeneity in short-run risk to produce cross-sectional variation in risk premia.

The specification of cash flows and informational assumptions pursued here is but one

[8] For example, Cochrane (2005), p. 18 writes “No model has yet been able to account for the equity premium with low risk aversion.”

of many that could be fruitfully studied in future work. Different cash flow configurations naturally lead to different assumptions about the information investors may have about those configurations. Going further, informational barriers may be compounded with uncertainty over the cash flow model itself, possibly leading investors to pursue robustness of the type studied by Anderson, Hansen and Sargent (1998), and Anderson, Hansen and Sargent (2003). Exploring these implications to their fullest suggests a fascinating but vast scope of inquiry for future analysis.

6 Appendix

6.1 Numerical Solution

6.1.1 Full Information, Constant Variances

Under Full Information, there is a single state variable, x_t. We discretize and bound its support by forming a grid of K points $\{x_1, x_2, ... \, x_K\}$ on the interval [-5V(x) +5V(x)]. We choose K to be odd so that the unconditional mean of the state x is the middle point of our grid.

We discretize also the distribution of a standardized normal random variable by forming a grid of equidistant points $\{\epsilon_1, \epsilon_2, ... \, \epsilon_I\}$ over the interval [-5 +5], imposing:

$$p_i = \frac{e^{-\epsilon_i^2/2}}{\sum_1^I e^{-\epsilon_i^2/2}}, \; i = 1, 2, ...I$$

Again, we choose I to be odd so that $\epsilon_{(I-1)/2+1} = 0$.

Rewrite the Euler equations for the price-consumption ratio as:

$$\begin{aligned} W_c(x_k) &= \left(\sum_{i=1}^{I} \sum_{j=1}^{I} \delta^\theta e^{(1-\gamma)(\mu + x_k + \sigma\epsilon_i)} [1 + W_c(x'_{j|k})]^\theta p_i p_j \right)^{\frac{1}{\theta}} \\ x'_{j|k} &= \rho x_k + \sigma \varphi_x \epsilon_j \\ k &= 1, 2, ..., K, \end{aligned} \tag{13}$$

where $W_c(x_k)$ is the price-consumption ratio as a function of x in state k. The functional in (13) can be solved by noting that its right hand side is a contraction and treating $W_c(x)$ as the fixed point of $W_{c,n+1}(x) = T(W_{c,n}(x))$.

Approximate $W_{c,n}$ by a third order polynomial in x, and impose:

$$W_{c,n}(x'_{j|k}) = [1 \; x'_{j|k} \; (x'_{j|k})^2 \; (x'_{j|k})^3][\beta_{1,n} \; \beta_{2,n} \; \beta_{3,n} \; \beta_{4,n}]'$$

where the operator is initialized with an initial guess on the parameters β_0. Compute $W_{c,1}(x_k)$ for every $x_k \in \{x_1, x_2, ... \, x_K\}$, and stack the resulting values in the vector $\overrightarrow{W}_{c,1} \in R^K$. Using least squares the guesses are updated: $\beta_1 = (\Upsilon'\Upsilon)^{-1}\Upsilon'\overrightarrow{W}_{c,1}$, where:

$$\Upsilon = \begin{bmatrix} 1 & x_1 & (x_1)^2 & (x_1)^3 \\ 1 & x_2 & (x_2)^2 & (x_2)^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_k & (x_k)^2 & (x_k)^3 \\ & & & \end{bmatrix}$$

We repeat these steps until convergence (tolerance level = .1e-5).

Once $W_c(x) = [1\ x\ x^2\ x^3]\beta$ has been found, the stochastic discount factor has the following expression:

$$M_{k,i,j} = \delta^\theta e^{-\gamma(\mu+x_k+\sigma\epsilon_i)} \left(\frac{1+W_c(\rho x_k + \sigma\varphi_x\epsilon_j)}{W_c(x_k)}\right)^{\theta-1}$$

price-dividend ratios are found in a similar way by iterating until convergence the following recursion:

$$\begin{aligned} W_{d,n+1}(x_k) &= \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{l=1}^{I}\delta^\theta e^{-\gamma(\mu+x_k+\sigma\epsilon_{c,i})}\left(\frac{1+W_c(x'_{j|k})}{W_c(x_k)}\right)^{\theta-1}\times \\ &\quad \times[1+W_{d,n}(x'_{j|k})]e^{(\mu+\phi_x x_k+\phi_c\sigma\epsilon_{c,i}+\sigma\varphi_d\epsilon_{d,l})}p_ip_jp_l \qquad (14) \\ \\ W_{d,n}(x'_{j|k}) &= [1\ x'_{j|k}\ (x'_{j|k})^2\ (x'_{j|k})^3]\beta_{d,n} \end{aligned}$$

The coefficients of the polynomial expansion for the price-dividends are updated by the following OLS formula: $\beta_{d,n+1} = (\Upsilon'\Upsilon)^{-1}\Upsilon'\overrightarrow{W}_{d,n+1}$.

For $n \to \infty$, $\beta_{d,n+1} \to \beta_d = (\Upsilon'\Upsilon)^{-1}\Upsilon'\overrightarrow{W}_d$.

To solve for zero coupon equity price-dividend Ratios note the following equivalence holds:

$$\begin{aligned} W_{d,t} &= \sum_{n=1}^{\infty} W_{d,t}^n \qquad (15) \\ &\text{where} \\ W_{d,t}^0 &\equiv 1 \\ W_{d,t}^n &= E_t\left[e^{m_{t+1}+\Delta d_{t+1}}W_{d,t+1}^{n-1}\right],\ n=1,2,... \end{aligned}$$

Implement the following recursion across maturities:

$$\begin{aligned}
W_d^n(x_k) &= \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{l=1}^{I}\delta^\theta e^{-\gamma(\mu+x_k+\sigma\epsilon_i)}\left(\frac{1+W_c(x'_{j|k})}{W_c(x_k)}\right)^{\theta-1}\times \\
&\quad \times[W_d^{n-1}(x'_{j|k})]e^{(\mu+\phi_x x_k+\phi_c\sigma\epsilon_i+\sigma\varphi_d\epsilon_l)}p_i p_j p_l
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
k &= 1,2,...,K \\
W_d^{n-1}(x'_{j|k}) &= [1\ x'_{j|k}\ (x'_{j|k})^2\ (x'_{j|k})^3][\beta_1^{n-1}\ \beta_2^{n-1}\ \beta_3^{n-1}\ \beta_4^{n-1}]' \\
\beta^{n-1} &= (\Upsilon'\Upsilon)^{-1}\Upsilon'\overrightarrow{W}_d^{n-1}\ \ n=2,3,.... \\
\beta^0 &\equiv [0\ 0\ 0\ 0]'
\end{aligned}$$

$$\lim_{n\to\infty}\sum_{j=1}^{n}\beta^{n-1}=\beta_d$$

This amounts to a sequence of quadrature problems that have to be solved recursively since the price of the asset with maturity n depends on the price of the asset with maturity $n-1$.

6.1.2 Limited Information, Constant Variances

In Limited Information, the Price-Consumption Ratio and the stochastic discount factor depend just on one relevant state: $\widehat{x}$, here denoted $\widehat{\Delta c}$. We discretize and bound its support by forming a grid of K points $\{\widehat{\Delta c}_1, \widehat{\Delta c}_2, \ldots \widehat{\Delta c}_K\}$ on the interval [-5$V(\widehat{\Delta c})$ +5$V(\widehat{\Delta c})$]. We choose K to be odd so that the unconditional mean of the state $\widehat{\Delta c}$ is the middle point of our grid, $\widehat{\Delta c}_t \perp v_{c,t+1}$.

The Euler equation for the Price-Consumption ratio is:

$$W_c(\widehat{\Delta c}_k) = \left(\sum_{j=1}^{I}\delta^\theta e^{(1-\gamma)(\mu+\widehat{\Delta c}_k+\sigma_{v_c}\epsilon_j)}[1+W_c(\widehat{\Delta c}'_{j|k})]^\theta p_j\right)^{\frac{1}{\theta}} \tag{17}$$

where

$$\widehat{\Delta c}'_{j|k} = \rho\widehat{\Delta c_k}+(\rho-b_c)\sigma_{v_c}\epsilon_j$$

solved by iterating until convergence the following recursion:

$$\begin{aligned}
W_{c,n}(\widehat{\Delta c}_k) &= \left(\sum_{j=1}^{I}\delta^\theta e^{(1-\gamma)(\mu+\widehat{\Delta c}_k+\sigma_{v_c}\epsilon_j)}[1+W_{c,n-1}(\widehat{\Delta c}'_{j|k})]^\theta p_j\right)^{\frac{1}{\theta}} \\
n &= 1,2,...
\end{aligned}$$

where the function is interpolated by a third order polynomial in $\widehat{\Delta c}$ such that:

$$\begin{aligned}
W_{c,n-1}(x'_{j|k}) &= [1\ \widehat{\Delta c}'_{j|k}\ (\widehat{\Delta c}'_{j|k})^2\ (\widehat{\Delta c}'_{j|k})^3][\beta_{1,n-1}\ \beta_{2,n-1}\ \beta_{3,n-1}\ \beta_{4,n-1}]' \\
\beta_n &= (\Phi'\Phi)^{-1}\Phi'\overrightarrow{W}_{c,n} \text{ n} = 1,\ 2,\ 3,... \\
&\text{where} \\
\Phi &= \begin{bmatrix} 1 & \widehat{\Delta c}_1 & (\widehat{\Delta c}_1)^2 & (\widehat{\Delta c}_1)^3 \\ 1 & \widehat{\Delta c}_2 & (\widehat{\Delta c}_2)^2 & (\widehat{\Delta c}_2)^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \widehat{\Delta c}_k & (\widehat{\Delta c}_k)^2 & (\widehat{\Delta c}_k)^3 \\ & & & \end{bmatrix} \\
\beta_0 &: \text{initial guess}
\end{aligned}$$

The price-dividend ratio is a function of the state variable $\widehat{x}_d \equiv \widehat{\Delta d}$ and the shock v_d:

$$\begin{aligned}
\begin{bmatrix} v_{c,t+1} \\ v_{d,t+1} \end{bmatrix} &\sim i.i.d.N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{v_c} & \sigma_{v_c,c_d} \\ \sigma_{v_c,v_d} & \sigma_{v_d^2} \end{bmatrix}\right) \\
&\text{and} \\
\begin{bmatrix} \widehat{\Delta c} \\ \widehat{\Delta d} \end{bmatrix} &\sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{\widehat{\Delta c}} & \sigma_{\widehat{\Delta c},\widehat{\Delta d}} \\ \sigma_{\widehat{\Delta c},\widehat{\Delta d}} & \sigma^2_{\widehat{\Delta d}} \end{bmatrix}\right)
\end{aligned}$$

- A grid of combinations $(\widehat{\Delta d}_{g|k}, \widehat{\Delta c}_k)$ is stacked in a matrix S with dimension $(K \times G) \times 2$:

$$S = \begin{bmatrix} \widehat{\Delta c}_1 & \widehat{\Delta d}_{1|1} \\ \widehat{\Delta c}_1 & \widehat{\Delta d}_{2|1} \\ \vdots & \vdots \\ \widehat{\Delta c}_1 & \widehat{\Delta d}_{g|1} \\ \widehat{\Delta c}_2 & \widehat{\Delta d}_{1|2} \\ \vdots & \vdots \\ \widehat{\Delta c}_K & \widehat{\Delta d}_{g|K} \end{bmatrix}$$

The recursion used to find the price-dividend ratio is given by:

$$\begin{aligned}
W_{d,n}(\widehat{\Delta c}_s, \widehat{\Delta d}_s) &= \sum_{j=1}^{I}\sum_{i=1}^{I}\delta^\theta e^{-\gamma(\mu+\widehat{\Delta c}_s+\sigma_{v_c}\epsilon_j)}\left(\frac{1+V_c(\widehat{\Delta c}'_{j|s})}{V_c(\widehat{\Delta c}_s)}\right)^{\theta-1} \times \\
&\quad \times[1+W_{d,n-1}(\widehat{\Delta c}'_{j|s}, \widehat{\Delta d}'_{i|s})]e^{\mu+\widehat{\Delta d}_s+\sigma_{v_d}\epsilon_i}p_{ij} \\
(\widehat{\Delta c}_s, \widehat{\Delta d}_s) &= [S_{s,1}S_{s,2}] \\
s &= 1, 2, ..., K \times G
\end{aligned}$$

The price-dividend ratio is interpolated as above by a quadratic polynomial in the two states:

$$\begin{aligned}
W_{d,n-1}(\widehat{\Delta c}_s, \widehat{\Delta d}_s) &= [1\ \widehat{\Delta c}'_{j|k}\ \widehat{\Delta d}'_{i|k}\ (\widehat{\Delta c}'_{j|k})^2\ (\widehat{\Delta d}'_{i|k})^2\ \widehat{\Delta c}'_{j|k}\widehat{\Delta d}'_{i|k}] \times \\
&\quad \times[\beta^d_{1,n-1}\ \beta^d_{2,n-1}\ \beta^d_{3,n-1}\ \beta^d_{4,n-1}\ \beta^d_{5,n-1}\ \beta^d_{6,n-1}]' \\
\beta^d_n &= (\Phi^{d'}\Phi^d)^{-1}\Phi^{d'}\overrightarrow{W}_{d,n} \\
n &= 1, 2, 3, ... \\
&\text{where} \\
\Phi^d &= \begin{bmatrix} 1 & S_{1,1} & S_{1,2} & S^2_{1,1} & S^2_{1,2} & S_{1,1}S_{1,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & S_{G\times K,1} & S_{G\times K,2} & S^2_{G\times K,1} & S^2_{G\times K,2} & S_{G\times K,1}S_{G\times K,2} \end{bmatrix} \\
\beta_0 &: \text{initial guess}
\end{aligned}$$

Foe zero coupon equity price-dividends, we implement the following recursion:

$$\begin{aligned}
W^n_d(\widehat{\Delta c}_s, \widehat{\Delta d}_s) &= \sum_{j=1}^{I}\sum_{i=1}^{I}\delta^\theta e^{-\gamma(\mu+\widehat{\Delta c}_s+\sigma_{v_c}\epsilon_j)}\left(\frac{1+V_c(\widehat{\Delta c}'_{j|s})}{V_c(\widehat{\Delta c}_s)}\right)^{\theta-1} \times \qquad (18) \\
&\quad \times W^{n-1}_d(\widehat{\Delta c}'_{j|s}, \widehat{\Delta d}'_{i|s})e^{\mu+\widehat{\Delta d}_s+\sigma_{v_d}\epsilon_i}p_{ij} \\
W^{n-1}_d(\widehat{\Delta c}'_{j|s}, \widehat{\Delta d}'_{i|s}) &= [1\ \widehat{\Delta c}'_{j|k}\ \widehat{\Delta d}'_{i|k}\ (\widehat{\Delta c}'_{j|k})^2\ (\widehat{\Delta d}'_{i|k})^2\ \widehat{\Delta c}'_{j|k}\widehat{\Delta d}'_{i|k}] \times \\
&\quad \times[\beta^{n-1}_1\ \beta^{n-1}_2\ \beta^{n-1}_3\ \beta^{n-1}_4\ \beta^{n-1}_5\ \beta^{n-1}_6]' \\
\beta^n_d &= (\Phi^{d'}\Phi^d)^{-1}\Phi^{d'}\overrightarrow{W}^n_d \\
n &= 1, 2, 3, ... \\
\beta^0_d &= [0\ 0\ 0\ 0\ 0\ 0].
\end{aligned}$$

6.2 Cash Flow Betas

Table A.1 shows the output from regressions of dividend growth on 4 and 8 quarter trailing averages of consumption growth, using simulated data for cash flow models of the form (4)-(6). The slope coefficients in these regressions are denoted φ, and are reported for four models that vary only by the short-run risk exposure parameter ϕ_c. The model is

$$\Delta d_{t+1} = \alpha + \varphi \left(\frac{1}{K} \sum_{i=1}^{K} \Delta c_{t+1-i} \right) + \varepsilon_{t+1}.$$

The model is simulated at a monthly frequency, consumption and dividend data are time-aggregated to quarterly frequency, and regressions run on quarterly data, as in Bansal et al. (2006). The results for one parameter configuration are displayed in Table A.1, but findings for other parameter configurations studied in the main text are similar. The Table shows that heterogeneity in exposure to short-run consumption risk can generate heterogeneity in cash flow betas φ, when the cash flow betas are constructed from $K = 4$ and $K = 8$ quarter trailing moving averages of consumption growth. This occurs only when the data are time-averaged; regressions on monthly data produce no such discernable spread in cash flow betas across assets that differ solely by ϕ_c. The reason is that time-averaging introduces additional serial correlation into the growth rates of consumption and dividends. The overlapping nature of the time-aggregate data therefore generates a correlation between dividend growth and lagged consumption growth that rises with the sensitivity of dividend growth to consumption risk that is i.i.d. at the monthly frequency (but not at the time-aggregate quarterly frequency). The longer the horizon K, the smaller is this affect.

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Table 1: Asset Pricing Implications of Limited Information

		$E(P/D)$		$E(r_i - r_f)$		$E(r_f)$		$\sigma(r_i)$		$\sigma(r_f)$	
Row	Model	FI	LI	FI	LI	FI	LI	FI	LI	FI	LI
1	$\phi_x = 1 \quad \phi_c = 2.2$	111	300	1.06	1.20	1.65	0.95	17.28	17.43	1.18	0.74
2	$\phi_x = 1 \quad \phi_c = 6$	40	14	2.66	7.73	1.66	0.95	22.97	22.44	1.18	0.74
3	$\phi_x = 2 \quad \phi_c = 2.2$	32	300	3.37	1.26	1.65	0.95	18.62	19.95	1.18	0.74
4	$\phi_x = 2 \quad \phi_c = 6$	22	14	4.80	8.12	1.65	0.95	23.80	23.92	1.18	0.74
5	$\phi_x = 3 \quad \phi_c = 2.2$	20	238	5.15	1.26	1.65	0.95	20.53	23.29	1.18	0.74
6	$\phi_x = 3 \quad \phi_c = 6$	16	13	6.40	8.42	1.65	0.95	25.21	26.09	1.18	0.74

Notes: This table financial statistics of the model with full information (FI) and limited information (LI) for varying degrees of exposure to the long-run and short-run risk components, governed by ϕ_x and ϕ_c, respectively. The other parameters are set to $\gamma = 10$, $\psi = 1.5$, $\delta = 0.998985$, $\mu = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\sigma_{\varepsilon_z} = 0.044$, $\sigma_{\varepsilon_d} = 6$. $E(r_i - r_f)$ denotes the annual log risk-premium, in percent; $E(r_f)$ denotes the annual log risk-free rate, in percent, and $\sigma(r_i)$ and $\sigma(r_f)$ denote the standard deviations of the annual equity return and risk-free rate, respectively. $E(P/D)$ is the annual price-dividend ratio. Statistics are averages from 1000 simulated samples of 840 monthly observations.

Table 2: Asset Pricing Implications of Limited Information

Row	Model		$E(P/D)$		$E(r_i - r_f)$		$E(r_f)$		$\sigma(r_i)$		$\sigma(r_f)$	
			FI	LI	FI	LI	FI	LI	FI	LI	FI	LI
1	$\phi_x = 1$	$\phi_c = 2.5$	17	35	4.19	3.12	3.41	1.61	18.13	20.34	3.05	2.48
2	$\phi_x = 1$	$\phi_c = 4$	14	8	5.30	12.33	3.41	1.61	19.41	20.83	3.05	2.48
3	$\phi_x = 1.5$	$\phi_c = 2.5$	9	21	9.48	4.96	3.41	1.61	19.20	23.37	3.05	2.48
4	$\phi_x = 1.5$	$\phi_c = 3$	9	12	9.99	8.28	3.41	1.61	19.61	23.05	3.05	2.48
5	$\phi_x = 1.5$	$\phi_c = 4$	8	7	10.43	14.41	3.41	1.61	20.87	23.16	3.05	2.48
6	$\phi_x = 2$	$\phi_c = 2.5$	7	15	12.87	6.67	3.41	1.61	20.63	26.86	3.05	2.48
7	$\phi_x = 2$	$\phi_c = 4$	6	6	14.35	16.60	3.41	1.61	22.09	25.76	3.05	2.48

Notes: This table financial statistics of the model with full information (FI) and limited information (LI) for varying degrees of exposure to the long-run and short-run risk components, governed by ϕ_x and ϕ_c, respectively. The other parameters are set to $\gamma = 15$, $\psi = 1.3$, $\delta = 0.993$, $\mu = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\sigma_{\varepsilon_x} = 0.1$, $\sigma_{\varepsilon_d} = 6$. $E(r_i - r_f)$ denotes the annual log risk-premium, in percent; $E(r_f)$ denotes the annual log risk-free rate, in percent, and $\sigma(r_i)$ and $\sigma(r_f)$ denote the standard deviations of the annual equity return and risk-free rate, respectively. $E(P/D)$ is the annual price-dividend ratio. Statistics are averages from 1000 simulated samples of 840 monthly observations.

Table 3: Limited Imformation Models of Value and Growth Portfolios Based on Shares

	G	Growth to Value								V	V-G
Portfolio	1	2	3	4	5	6	7	8	9	10	10-1
$E\left(R^i - R^f\right)$	7.13	7.15	7.21	7.34	7.59	8.99	10.26	10.80	11.18	11.40	4.27
Sharpe Ratio	0.32	0.32	0.32	0.33	0.34	0.36	0.40	0.46	0.50	0.51	0.20
CAPM: $R_t^i - R_t^f = \alpha_i + \beta_i\left(R_t^m - R_t^f\right) + \varepsilon_{it}$											
α_i	-1.52	-1.50	-1.43	-1.31	-1.04	-0.52	0.41	1.72	2.66	2.93	4.45
β_i	1.01	1.01	1.01	1.01	1.01	1.00	1.00	0.99	0.99	0.98	-0.03
CAPM & HML: $R_t^i - R_t^f = \alpha_i + \beta_i\left(R_t^m - R_t^f\right) + \gamma_i HML_t + \varepsilon_{it}$											
α_i	0.06	0.06	0.06	0.06	0.07	0.08	0.11	0.18	0.27	0.06	0.00
β_i	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
γ_i	-0.35	-0.35	-0.34	-0.31	-0.25	-0.13	0.07	0.35	0.54	0.65	1.00

Notes: In each simulation month, firms are sorted into deciles based on the price-dividend ratio. Returns are calculated over the subsequent year and reported annualized in percent. The parameter values are the same as those in Table 2 where the market portfolio has $\phi_x = 1$ and $\phi_c = 3$.

Table 4: Limited Imformation Models of Value and Growth Portfolios Based on Shares

	G				Growth to Value					V	V-G
Portfolio	1	2	3	4	5	6	7	8	9	10	10-1
$E\left(R^i - R^f\right)$	7.38	7.41	7.47	7.61	7.89	8.44	9.43	10.80	11.82	12.07	4.70
Sharpe Ratio	0.31	0.31	0.32	0.32	0.34	0.36	0.40	0.47	0.51	0.52	0.21
CAPM: $R_t^i - R_t^f = \alpha_i + \beta_i\left(R_t^m - R_t^f\right) + \varepsilon_{it}$											
α_i	-1.69	-1.66	-1.59	-1.45	-1.16	-0.59	0.44	1.88	2.81	3.19	4.88
β_i	1.01	1.01	1.01	1.01	1.01	1.00	1.00	0.99	0.99	0.98	-0.02
CAPM & HML: $R_t^i - R_t^f = \alpha_i + \beta_i\left(R_t^m - R_t^f\right) + \gamma_i HML_t + \varepsilon_{it}$											
α_i	0.05	0.06	0.06	0.059	0.06	0.08	0.11	0.19	0.28	0.05	0.00
β_i	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
γ_i	-0.36	-0.35	-0.34	-0.31	-0.25	-0.14	0.07	0.35	0.54	0.64	1.00

Notes: In each simulation month, firms are sorted into deciles based on the price-dividend ratio. Returns are calculated over the subsequent year and reported annualized in percent. The parameter values are the same as those in Table 2 where the market portfolio has $\phi_x = 1$ and $\phi_c = 3$, with the following exceptions: $\gamma = 16$, $\psi = 1.25$, $\delta = 0.992$.

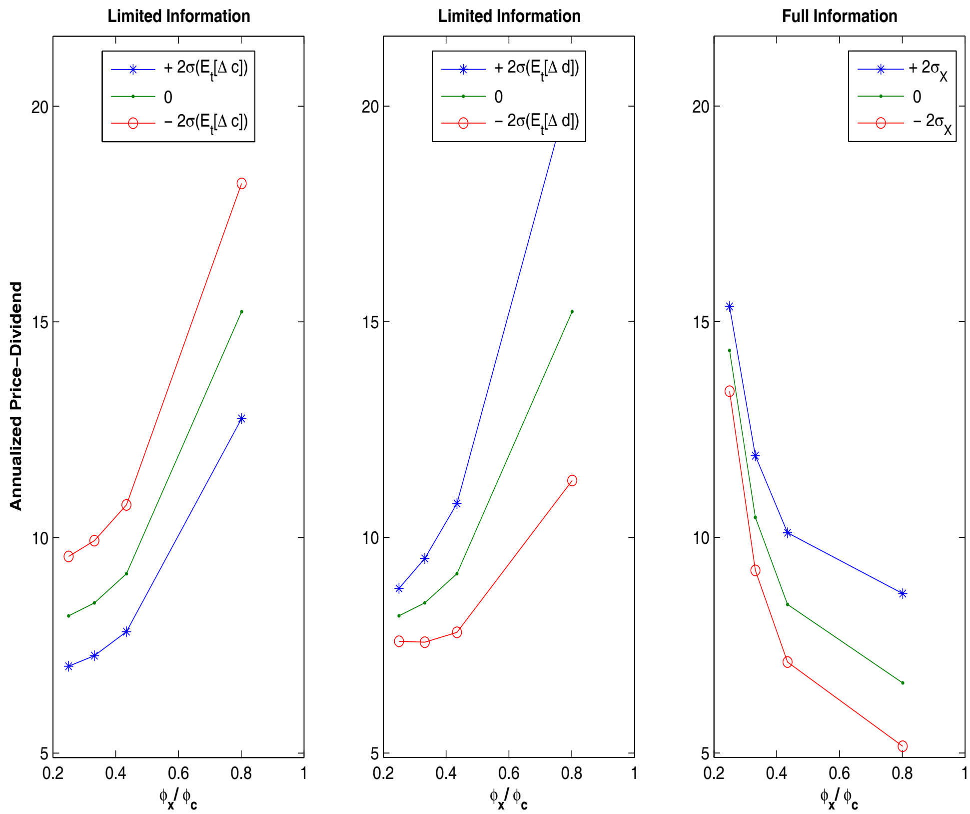
Table A1: Cash Flow Betas

Regression: $\Delta d_{t+1} = \alpha + \varphi \left(\frac{1}{K} \sum_{i=1}^{K} \Delta c_{t+1-i} \right) + \varepsilon_{t+1}$

		$K = 4$		$K = 8$	
		φ	t-stat	φ	t-stat
$\phi_x = 3$	$\phi_c = 0.5$	0.96	1.69	1.23	1.71
$\phi_x = 3$	$\phi_c = 3$	1.19	1.94	1.37	1.76
$\phi_x = 3$	$\phi_c = 6$	1.45	1.95	1.52	1.61
$\phi_x = 3$	$\phi_c = 10$	1.80	1.81	1.73	1.36

Notes: This table displays regression coefficients and t-statistics from regressions of quarterly dividend growth on to smoothed consumption growth. The quarterly data are time-aggregated from monthly data. The reported statistics are averages from 1000 simulations of length 1000 months (250 quarters). The other parameters are set to $\mu = 0.0015$, $\rho = 0.979$, $\sigma = 0.0078$, $\sigma_{\varepsilon_z} = 0.044$, $\sigma_{\varepsilon_d} = 6$.

Figure 1: Price-Dividend Ratios



Notes: This figure displays price-dividend ratios at steady state, and plus/minus two standard deviations of the state variables(s) around steady state, as a function of the relative exposure to long-run risk, governed by ϕ_x, and to short-run risk, governed by ϕ_c. Held fixed is the five-quarter variance of dividend growth attributable to the consumption innovations. Other parameters are calibrated as in Table 2.

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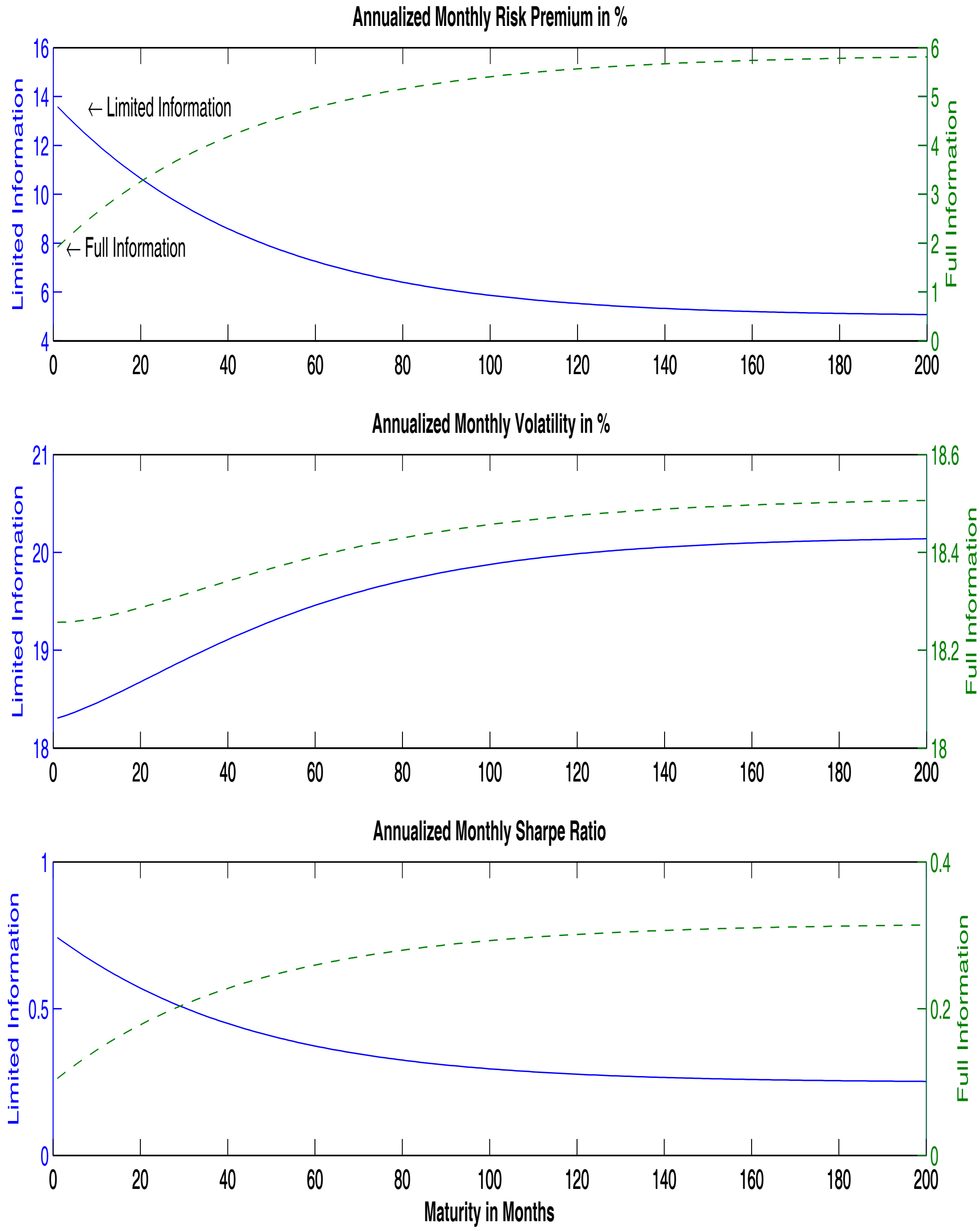
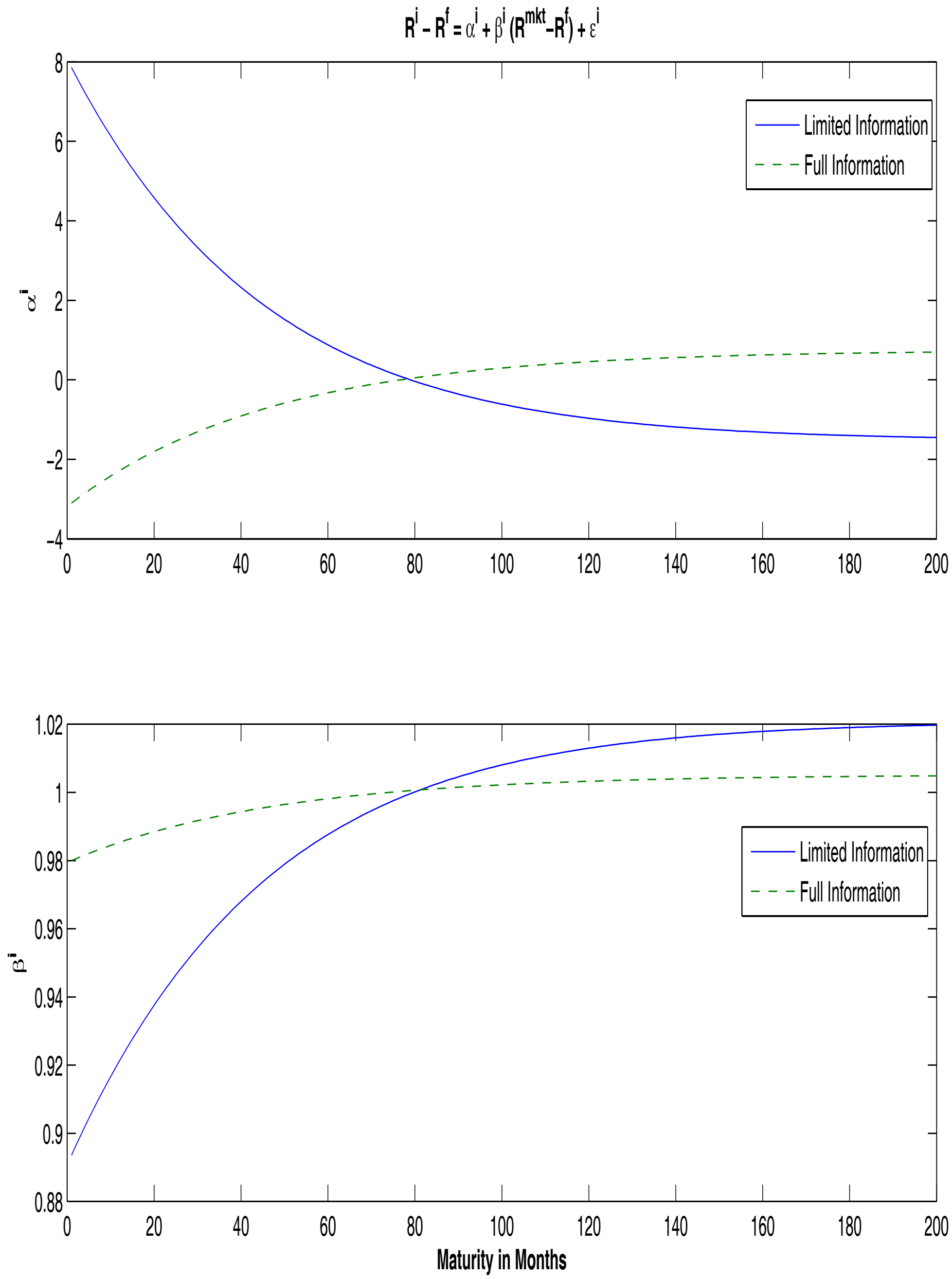
Figure 2: Zero-Coupon Equity



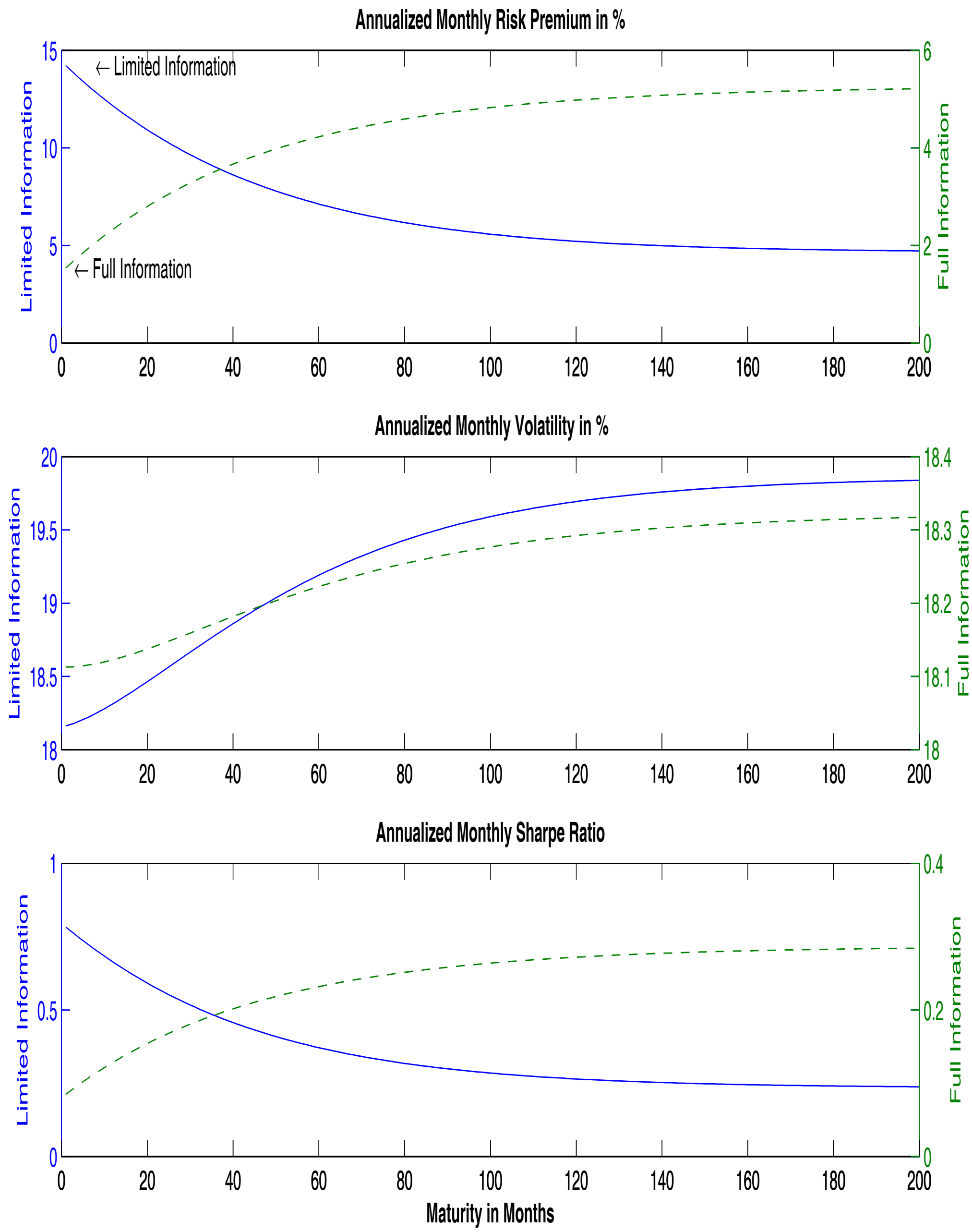
Notes: The top panel shows the risk-premia on zero-coupon equity over the risk-free rate as a function of maturity in months; the middle panel shows the standard deviation of returns on zero-coupon equity; the bottom panel shows the Sharpe ratio. Returns are simulated at a monthly frequency and aggregated to annual frequency. Parameters are calibrated as in Table 2 with $\phi_x = 1$ and $\phi_c = 3$.

Table 3: CAPM Regressions for Zero-Coupon Equity



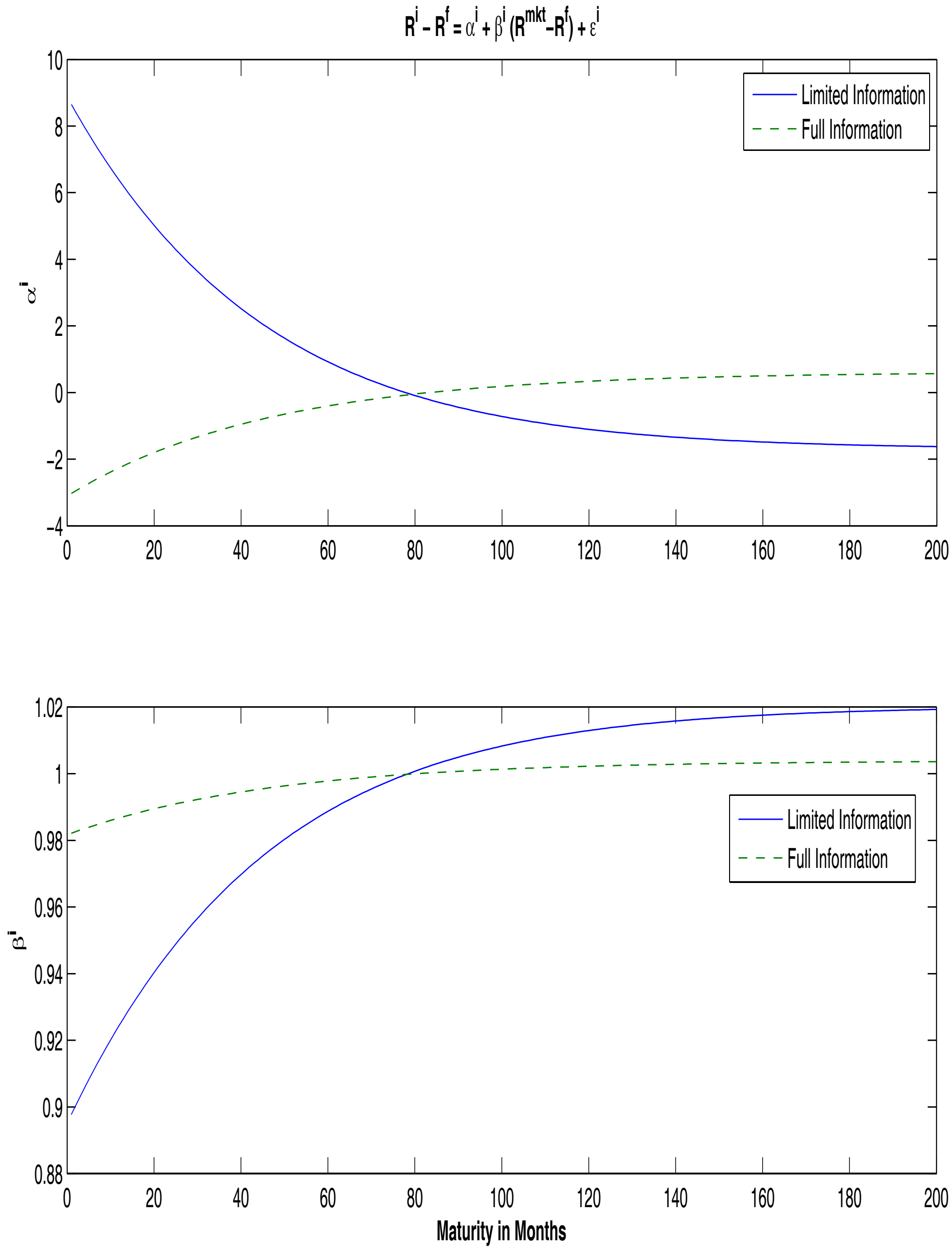
Notes: The top panel shows the intercept from regressions of zero-coupon equity excess returns on the excess return of the market, as a function of maturity in months; the bottom panel shows the slope coefficient from the same regression. Returns are simulated at a monthly frequency and aggregated to annual frequency. Parameters are calibrated as in Table 2 with $\phi_x = 1$ and $\phi_c = 3$.

Table 4: Zero-Coupon Equity



Notes: The top panel shows the risk-premia on zero-coupon equity over the risk-free rate as a function of maturity in months; the middle panel shows the standard deviation of returns on zero-coupon equity; the bottom panel shows the Sharpe ratio. Returns are simulated at a monthly frequency and aggregated to annual frequency. Parameters are calibrated as in Table 2, with $\phi_x = 1$ and $\phi_c = 3$ and with the following exceptions: $\Psi = 1.25$, $\gamma = 16$, $\delta = 0.992$.

Figure 5: CAPM Regressions for Zero-Coupon Equity



Notes: The top panel shows the intercept from regressions of zero-coupon equity excess returns on the excess return of the market, as a function of maturity in months; the bottom panel shows the slope coefficient from the same regression. Returns are simulated at a monthly frequency and aggregated to annual frequency. Parameters are calibrated as in Table 2, with $\phi_x = 1$ and $\phi_c = 3$ and with the following exceptions: $\Psi = 1.25$, $\gamma = 16$, $\delta = 0.992$.