

Demographic Trends, Fiscal Policy and Trade Deficits

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Abstract

In this paper, I argue that demographic factors play a central role in accounting for the trade deficit experienced by the U.S. during the last three decades. The main idea is that cross-country demographic differentials lead to adjustments in savings and investments which are associated with international capital flows towards relatively young and rapidly growing economies. I develop a tractable two-country framework with life-cycle structure that formalizes this intuition. The model permits to illustrate analytically and quantitatively the contribution of demographic variables in determining the equilibrium trade balance. I show that persistent differences in population aging can explain a significant fraction of the negative trend in the U.S. trade balance. Notably, the explicit consideration of the demographic transition also helps to reconcile the dynamics of the trade balance with the evolution of the U.S. fiscal deficits and generates a declining pattern for the real interest rate broadly consistent with the data.

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1 Introduction and Related Literature

The dynamics of the U.S. external position have recently received a great deal of attention in the economic community. In 2004 the U.S. trade balance reached a record deficit of more than 5% of GDP. This figure is the result of a long run trend that is projected to continue, at least in the near future.

The objective of this paper is to highlight the central role played by demographic factors in explaining the U.S. trade deficit during the last three decades. The results of this work demonstrate that demographic differentials are crucial to:

1. Account for the negatively sloped trend of the U.S. trade balance.
2. Reconcile the dynamics of the trade balance with the evolution of fiscal deficits.
3. Capture the persistent decline of the world real interest rate.

The key driving force behind these findings is the adjustment of private savings to the demographic transition. In an economy with aging population, individuals plan for longer retirement periods by saving more during their working years. Moreover, during retirement, their consumption is spread over a longer period of time. If the economy is open and demographic profiles differ across countries, relative savings generate capital flows that are associated with variations in the stock of net foreign assets and trade imbalances.

For the purpose of this paper, I concentrate on the bilateral trade balance between the United States and the other six major industrialized countries of the world economy (Canada, France, Germany, Italy, Japan and the United Kingdom - henceforth, the G6). The reason for narrowing the focus on this subset of the U.S. trade partners is that the G6 feature an economic structure that is highly comparable to the U.S. under several respects. At the same time, however, the G6 display notable differences in terms of demographic variables, as I illustrate more thoroughly in the next section.

The left panel of Figure 1 shows that, at least until the late 1990s, the U.S. trade balance vis-a-vis the G6 accounted for the bulk (about 80% on average) of the total U.S. external imbalance.¹ The right panel of Figure 1 depicts the U.S. trade balance against the G6 together

¹The recent departure of the U.S. trade balance series vis-a-vis the rest of the world from the series vis-a-vis the G6 depends upon the capital flows towards the U.S. from oil producers, newly developed and emerging market economies. This aspect of the U.S. external position (the “global saving glut”, in the words of Bernanke [2005]) is not the focus of this paper. See Caballero, Fahri and Gourinchas [2005] for an explanation of this phenomenon based on the inability of emerging market economies to generate enough reliable saving instruments after the Asian crises of the late 1990s.

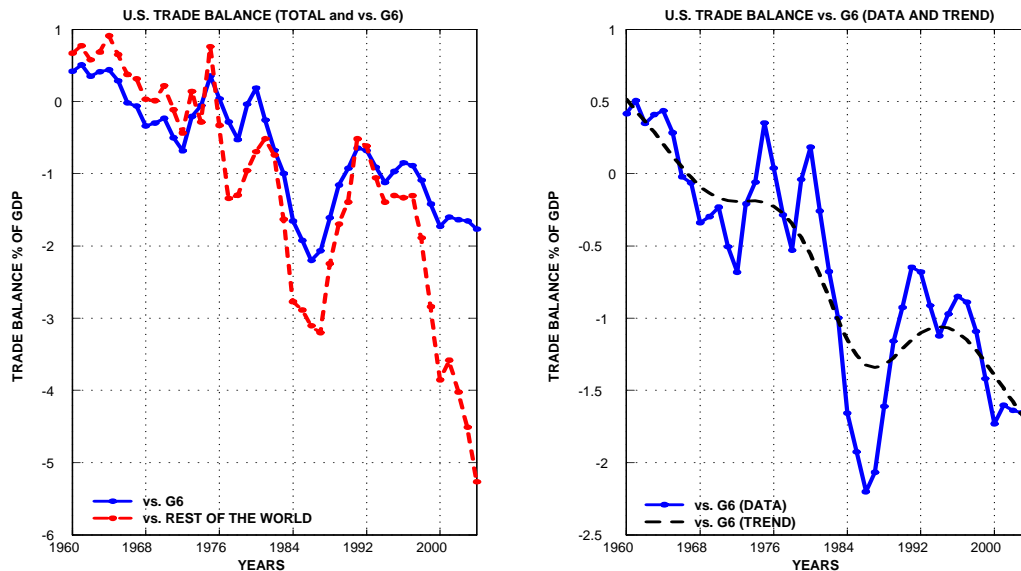


Figure 1: THE U.S. TRADE BALANCE.

with its long run trend. The picture clearly identifies the overall decline of the long run trend and the negative sign which characterizes large part of the sample.

Recent contributions (Caballero, Fahri and Gourinchas [2005] and Engel and Rogers [2005]) have emphasized the role of productivity growth differentials in explaining the U.S. external deficits. An explanation of the dynamics of the U.S. trade deficit based on productivity differentials requires not only current productivity growth to be higher in the U.S. than in the rest of the world but also the expectations of future productivity differentials to be positive and increasing. In the words of Engel and Rogers [2005], “*the assumptions on future growth in U.S. shares [of world GDP] that can explain the level in 2004 would imply that the U.S. deficit should have been even larger in earlier years*”. Caballero, Fahri and Gourinchas [2005] resolve this issue by postulating permanent growth rate differentials between the U.S. and the G6. The data, however, hardly support this assumption. Figure 2 plots different measures of GDP growth rates for the U.S. and the G6 during the last twenty five years. While differences exist in various periods of the sample, and in some cases persist for few years, the hypothesis of permanent differences appears to be quite extreme.²

This work proposes an alternative explanation for the long run trend of the U.S. exter-

²Productivity growth rate differentials likely play a prominent role in accounting for the variations in the trade balance at higher frequencies.

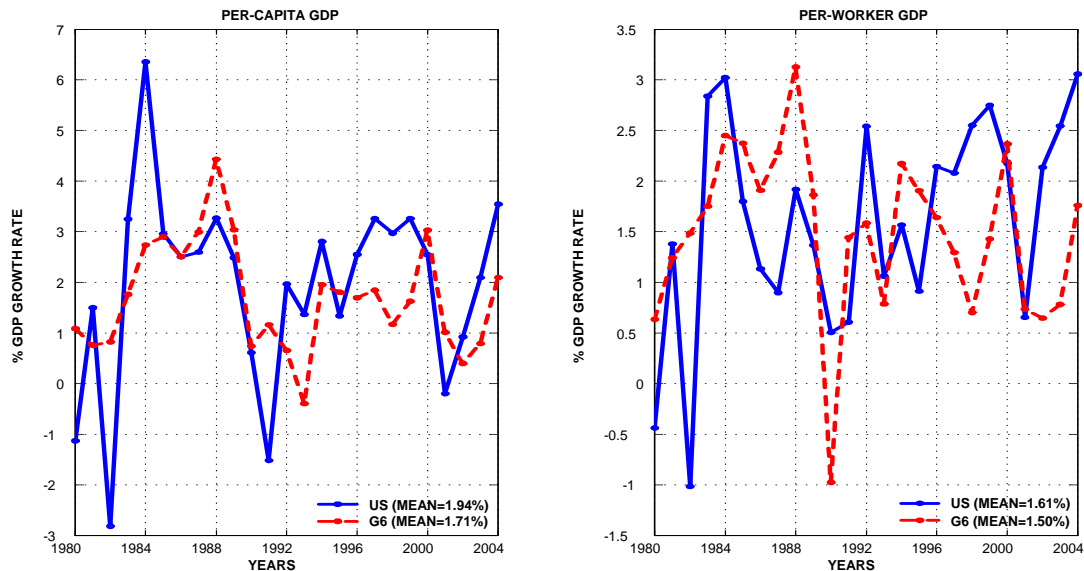


Figure 2: GDP GROWTH RATES.

nal position based on demographic factors. The methodological contribution of the paper is to develop a tractable open economy model with life-cycle structure that identifies the transmission mechanism and the quantitative importance of demographic variables for the determination of the equilibrium trade balance. The resulting framework shares the basic features of the neoclassical growth model. The only complication of considering life-cycle behavior and non-stationary demographics derives from the presence of two extra state variables that capture the distribution of wealth across cohorts and the ratio between the number of retirees and the number of workers.

The demographic structure assumed in this paper builds upon the formulation in Blanchard [1985] and Weil [1989] by adding a random transition from employment to retirement, an element of the life-cycle not present in those two studies. In Blanchard [1985], individuals face uncertainty about their lifetime horizon but the survival probability also governs the fertility rate. Hence, an increase in life expectancy also implies an increase in population growth so that the aggregate size of population is constant. On the other hand, in Weil [1989], households are infinitely lived but in each period a new cohort is born so that population grows over time. In this paper, the probability of surviving is decoupled from the population growth rate. It is then possible to isolate the contribution of smaller population growth rates and longer lifetime horizons in determining the equilibrium trade balance. The life-cycle transition

from the labor force into retirement generates a reasonable pattern for aggregate savings in response to demographic shocks. Saving differentials across countries indeed constitute the main driving force behind the adjustment.

The explicit link between demographics and external imbalances has been previously explored in the literature. Obstfeld and Rogoff [1996] discuss some of the basic issues in the context of a two-period life-cycle model. The main prediction of such a framework is that countries with higher population growth rates should display, *ceteris paribus*, higher saving rates. The two-period model necessarily treats the lifetime horizon as fixed (and known). This paper instead emphasizes the importance of variable life expectancy in shaping the individual consumption-savings decisions. Countries where individuals live on average longer are associated with higher savings rates. Since those countries also feature lower population growth rates, the results in this paper are largely consistent with the data and reverse the argument in the traditional life-cycle literature.³

The quantitative importance of cross-country demographic differentials on the trade balance has recently been stressed by Henriksen [2002] and Feroli [2003] in the context of an open economy computable overlapping generation (OLG) model.⁴ Domeij and Flóden [2004] also simulate an open economy OLG framework driven by demographic differentials and test the explanatory power of the data artificially generated by the calibrated model for the time series of the U.S. current account. Their findings are consistent with a significant effect of demographic variables on external imbalances. The model in this paper remains considerably more tractable than a standard OLG framework. Yet, it provides a framework that can be employed to perform a number of numerical experiments. After illustrating analytically how demographic factors impact the trade balance, I present a quantitative analysis that shows how demographic forces are crucial to account for the declining trend in the U.S. trade balance in the last three decades.

The one source of short run variation that I consider in this paper is fiscal policy. In the absence of voluntary bequests, the presence of life-cycle features breaks one of the assumptions at the core of Ricardian equivalence. Therefore, I can study the impact of fiscal policy on the trade balance in an environment with time-varying demographic differentials across countries. Fiscal deficits alone lead to modest trade deficits, as also recently suggested by

³Brooks [2003] considers a four-period multiregion overlapping generation model and shows that the current demographic structure should be associated with capital flows from the G7 towards developing countries. This result also depends crucially on the fixed lifetime horizon assumed in that paper.

⁴Both papers extend the seminal work of Auerbach and Kotlikoff [1987] to a two-country setup.

Erceg, Guerrieri and Gust [2005]. With stationary demographics, the evolution of fiscal policy in the U.S. and in the G6 during the 1990s would generate counterfactual implications for the dynamics of the U.S. external imbalance. Given the negative trend induced by demographic differentials, fiscal consolidations lead to smaller improvements of the trade balance compared to the deteriorations associated with fiscal expansions. The second contribution of the paper is to show that, when demographic factors are properly considered, it is possible to reconcile the evolution of fiscal deficits with the dynamics of the trade balance over the entire sample period.

Finally, the model naturally generates a decreasing pattern for the world interest rate, which is broadly consistent with the evidence in the data. The decline of the real rate is the direct consequence of the global excess of savings associated with population aging. This finding carries important implications for the persistence and sustainability of fiscal and external deficits.⁵ Low interest rates increase the attractiveness of debt and lower the burden of outstanding liabilities. In this respect, the model improves upon large strands of the existing literature which tends to ignore the dynamics of the real interest rate.⁶

In the next section, I present some evidence and projections about demographic differentials between the U.S. and the G6 and review the evolution of fiscal policy in the two regions. I then describe the structure of the economy and the equilibrium for the two-country model. The next two sections of the paper present the analytical and quantitative results. The description of the data and the details of the analytical derivations are confined to the appendix.

2 Evidence on Demographic Trends and Fiscal Policy

This section documents the evolution of the levels and differentials in demographic factors and fiscal stances between the U.S. and the G6.

As portrayed in the top-left panel of Figure 3, the U.S. and the G6 have experienced a very different evolution of population growth rates. The growth rate has been substantially constant (around 1%) in the U.S. ever since 1970 whereas it has progressively decreased in

⁵Gourinchas and Rey [2005] study the sustainability of the U.S. trade balance in light of the intertemporal approach to the current account and emphasize valuation effects associated to the rate of return. Bohn [2004] finds evidence in favor of sustainability of the U.S. fiscal policy based on a test of the intertemporal government budget constraint. The level of the interest rate plays a crucial role for both results.

⁶One exception is Caballero, Fahri and Gourinchas [2005], who also obtain a decreasing path for the real interest rate in their model.

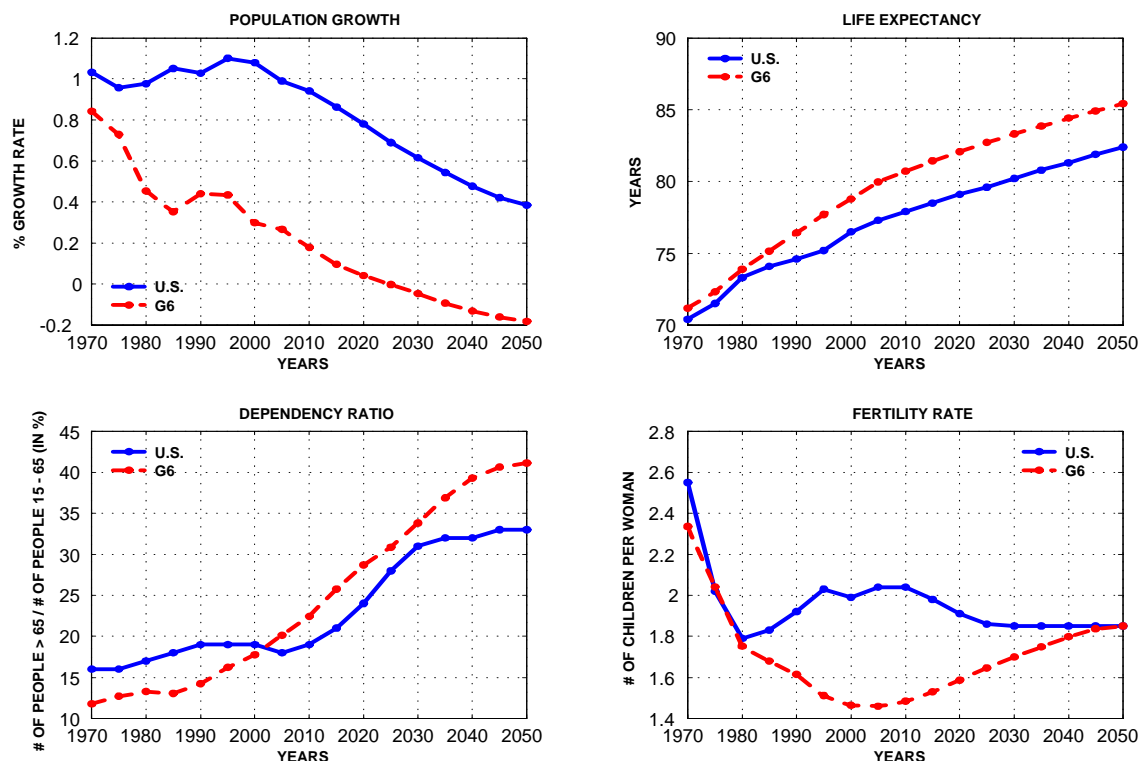


Figure 3: FOUR DEMOGRAPHIC INDICATORS.

the G6. Available projections point in the direction of a reduction in the U.S. growth rate starting in 2005. The declining trend is also expected to continue in the G6.

Population growth rates represent only one possible indicator of demographic trends. The top-right panel of Figure 3 reports the years of life expectancy for the U.S. and for the G6. The two bottom panels of Figure 3 respectively plot the dependency ratio and the fertility rates for the two economies. The dependency ratio is defined as the total number of individuals 65 years and older divided by the total number of individuals between 15 and 64 years of age. It represents a measure of dependency of the elderly on the fraction of the total population able to participate in the labor force.⁷ The fertility rates, the average number of children per woman in a given year, also confirm the overall declining demographic trends.

Two main points emerge clearly from this picture. First, lower population growth rates

⁷The dynamics of the dependency ratio play a very important role for public policy decisions concerning welfare programs. See, for instance, Attanasio, Kitao and Violante [2005] for an assessment of social security reforms in the light of the projected demographic trends.

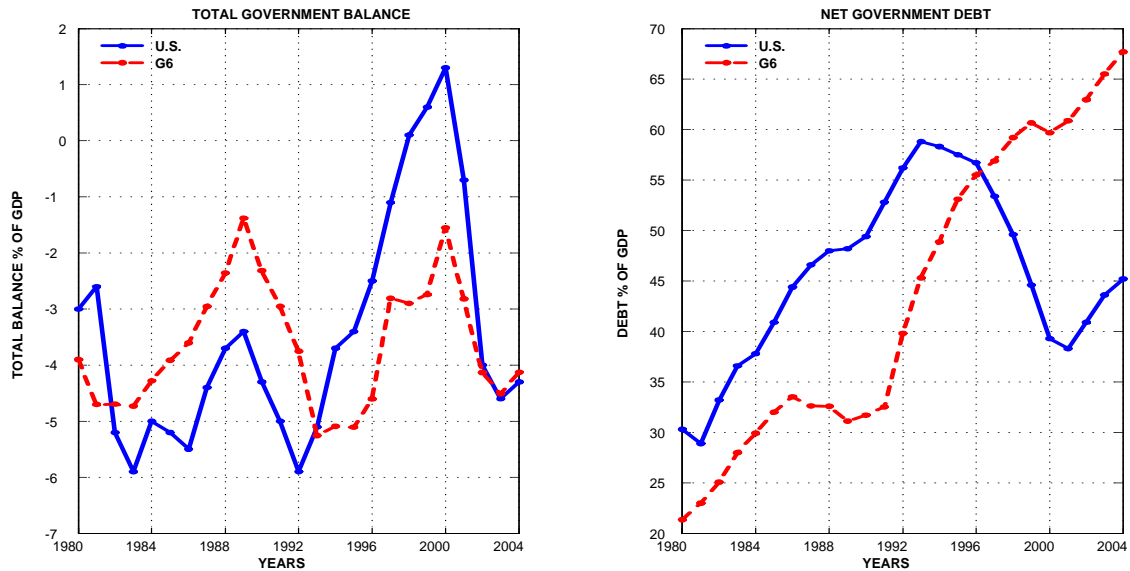


Figure 4: TWO MEASURES OF FISCAL POLICY.

and faster population aging is a phenomenon that concerns the U.S. as well as the G6. Second, the differentials in demographic indicators, which were quite small at the beginning of the 1970s, have become more pronounced during the last two decades and are projected to remain as such at least until 2050, with the G6 ahead of the U.S. by several decades. Both considerations turn out to be important for the quantitative analysis in this paper.

The second piece of evidence that I present in this section covers the evolution of the total government balance and the stock of net government debt from 1980 until 2004. As mentioned in the introduction, demographic trends are crucial in this paper to reconcile the idea that fiscal deficits entail external imbalances (the twin-deficits hypothesis) with the puzzling fact that the U.S. fiscal consolidation that took place in the 1990s was accompanied only by an initial small correction in the trade deficit followed by a fast deterioration thereafter. The key question in the debate about Ricardian equivalence and the real effects of fiscal policy is whether financing a certain level of spending with taxes or debt matters for aggregate consumption. For this reason, I concentrate on fiscal deficits and government debt rather than on government spending.⁸

Figure 4 reports the total government balance (left) and the net government debt (right)

⁸A constant ratio of government spending to GDP is also roughly consistent with the evidence for G7 countries.

for the U.S. and for the G6 as a fraction of their GDP levels. Both panels suggest a split of the sample in three parts. The first period corresponds to the 1980s and early 1990s. In those years, the U.S. was running a relatively expansionary fiscal policy. Fiscal deficits were on average about two percentage points higher than in the G6 and net government debt doubled to reach 60% in 1993. The second period roughly covers the 1990s. The main feature of that decade is the fiscal consolidation that took place in the U.S. and the increase in government debt in the G6. Beginning in 2000, fiscal policy in the U.S. has been again on an expansionary path relative to the G6. The rapid turnaround in the government balance has finally brought the U.S. deficit to the levels of the G6 at the end 2004.

The data highlight two regularities. First, fiscal stances follow similar patterns in the two regions, with two periods of expansion in the 1980s and during the last five years, spaced out by the consolidation of the 1990s. Second, the differentials for both deficits and debt levels appear to be small in magnitude. These two observations constitute the key aspects of fiscal policy for the following analysis.

3 A Life-Cycle Model of the U.S. and the G6

In this section, I set up a two-country world economy in which agents exhibit life-cycle behavior. An open economy represents the natural framework to think about the effects of demographics on the trade balance. A two-country model reveals particularly appropriate to the extent that the U.S. and the G6 are two major players in the world economy and is also instrumental to discuss the link between demographic factors and the real interest rate. Finally, the life-cycle structure not only naturally captures the key elements of the demographic transition but it also permits to consider non-Ricardian effects of fiscal policy.

The main difference with respect to the existing literature on computable life-cycle models concerns the tractability of the framework. I present a model in which the heterogeneity implicit in the life-cycle can be summarized by two state variables, the distribution of assets between retirees and workers and the ratio between the number of individuals in those two groups. This formulation adds life-cycle behavior on top of a basic structure close to the work of Blanchard [1985] and Weil [1989]. The framework is tractable enough to derive a closed form solution for aggregate consumption given prices, along the same lines of Gertler [1999]. This result is crucial to describe the basic mechanisms that underlie the effect of demographic factors on the trade balance.

The world is composed of two countries, Home (the U.S.) and Foreign (the G6), initially

identical in all respects. I describe in details the structure of the Home economy. Whenever necessary, I refer to Foreign variables with an asterisk “*”. At time t , the Home country is populated by a continuum of agents of mass N_t . The commodity space is constituted by a single consumption good which can be traded internationally at no shipping cost and serves as numeraire. There is no aggregate uncertainty. All agents have perfect foresight but can be surprised by unexpected exogenous shocks.

Individuals are born workers (denoted by the superscript w) and retire with probability ω , which I assume to be fixed throughout the analysis. Once a retiree (denoted by the superscript r), an individual survives from period t to period $t + 1$ with probability $\gamma_{t,t+1}$. The number of workers in period t is N_t^w . In each period $(1 - \omega + n_{t,t+1}) N_t^w$ individuals (workers) are born. The law of motion for the aggregate labor force is

$$N_{t+1}^w = (1 - \omega + n_{t,t+1}) N_t^w + \omega N_t^w = (1 + n_{t,t+1}) N_t^w, \quad (1)$$

so that $n_{t,t+1}$ represents the growth rate of the labor force between period t and $t + 1$. The number of retirees is denoted by N_t^r and evolves over time according to

$$N_{t+1}^r = (1 - \omega) N_t^w + \gamma_{t,t+1} N_t^r. \quad (2)$$

The ratio between the number of retirees and the number of workers, defined as $\psi_t \equiv N_t^r / N_t^w$, summarizes the heterogeneity in the population. The variable ψ_t is called “dependency ratio” and follows a law of motion that can be derived from expression [2] after dividing both sides by N_t^w and rearranging using [1]

$$(1 + n_{t,t+1}) \psi_{t+1} = (1 - \omega) + \gamma_{t,t+1} \psi_t. \quad (3)$$

The growth rate of the number of retirees can also be derived from expression [2]

$$\frac{N_{t+1}^r}{N_t^r} = \frac{1 - \omega}{\psi_t} + \gamma_{t,t+1} = (1 + n_{t,t+1}) \frac{\psi_{t+1}}{\psi_t}, \quad (4)$$

where the second equality makes use of [3]. Finally, total population grows according to

$$\frac{N_{t+1}}{N_t} = (1 + n_{t,t+1}) \frac{(1 + \psi_{t+1})}{(1 + \psi_t)}. \quad (5)$$

If population growth rates and surviving probabilities are constant, the dependency ratio is

also constant and equal to

$$\psi = \frac{1 - \omega}{1 + n - \gamma}. \quad (6)$$

The number of retirees relative to the number of worker depends positively on the probability of surviving and negatively on the probability of retirement and on the growth rate of the labor force. Under stationary demographics, from [4] and [5], it follows that the number of retirees N_t^r and the total population N_t grow at the same rate as the number of workers N_t^w , which is equal to n .

Workers supply inelastically one unit of labor and retirees do not work.⁹ Individual preferences belong to the recursive non-expected utility family, originally introduced by Kreps and Porteus [1978] and extended by Epstein and Zin [1989] to an infinite horizon framework. The general formulation for an individual of cohort $z = \{w, r\}$ reads as

$$V_t^z = \left\{ (C_t^z)^\rho + \beta_{t,t+1}^z [E_t (V_{t+1}^\alpha | z)]^\frac{\rho}{\alpha} \right\}^\frac{1}{\rho}, \quad (7)$$

where C_t^z denotes consumption and V_t^z stands for the value of utility in period t . The discount factor of a retiree differs from that of a worker in order to account for the probability of death

$$\beta_{t,t+1}^z = \begin{cases} \beta \gamma_{t,t+1} & \text{if } z = r \\ \beta & \text{if } z = w \end{cases}$$

The preferences in [7] feature the well-known property of separating intertemporal substitution from risk-aversion. The parameter ρ measures intertemporal substitution. In particular, the elasticity of intertemporal substitution is given by $\sigma \equiv 1/(1 - \rho)$. On the other hand, the parameter α captures risk aversion. More specifically, the coefficient of relative risk aversion is equal to $1 - \alpha$.

The expectation of V_{t+1} differs across cohorts because the future value of utility depends on the current employment status. In particular, a worker remains in the labor force with probability ω and retires with probability $1 - \omega$. The resulting expression for the expectation

⁹It is possible to relax both these assumptions without sacrificing the analytical tractability of the model. See Gertler [1999] for details.

of next period's value function is

$$E_t \{V_{t+1}^\alpha | z\} = \begin{cases} (V_{t+1}^r)^\alpha & \text{if } z = r \\ \omega (V_{t+1}^w)^\alpha + (1 - \omega) (V_{t+1}^r)^\alpha & \text{if } z = w \end{cases}$$

The fact that the probability of retirement and death are independent of age makes it possible to consider the problem of a generic individual within each cohort without specific reference to her age or to her retirement period. In what follows, I discuss two additional assumptions that keep the model analytically tractable and greatly facilitate aggregation.

The idiosyncratic uncertainty about the lifetime horizon is eliminated by assuming the existence of a perfect annuity market in which retirees can fully insure against the risk of death. Retirees turn their wealth over to a perfectly competitive mutual fund industry which invests the proceeds and pays back a premium over the market return to compensate for the probability of death.¹⁰ For a retiree who survives between period $t - 1$ and t , the return on a one dollar investment is $R_{W,t}/\gamma_{t-1,t}$, where $R_{W,t}$ is the world interest rate that clears the international capital markets. The extra-return over the market rate compensates for the probability of death.

The idiosyncratic uncertainty about employment tenure is limited by imposing that households are risk-neutral with respect to variations in income. To this extent, I set $\alpha = 1$ in [7]. For a given level of the interest rate, only average consumption tomorrow (appropriately weighted) matters for current consumption decisions.

The assumption of perfect insurance markets against the risk of death is quite common in the literature (see, for instance, Yaari [1965] and Blanchard [1985]). A similar mechanism to insure against labor income fluctuations would smooth consumption perfectly across work and retirement, hence shutting off the life-cycle channel which is at the very heart of the analysis. On the other hand, the assumption of risk neutrality, while still allowing for an arbitrary elasticity of substitution σ , mitigates the artificially high degree of labor income fluctuations induced by the constant transition probability into retirement.

I now turn to characterizing the consumption-savings problem for each cohort. I present the problem of a generic worker first and then the problem of a generic retiree.

¹⁰It is convenient to assume that in each country the mutual fund operates at the national level only. The reason is that, in the presence of cross-country differences in aging profile, no arbitrage would imply equalization of returns in the insurance market but not in the capital market. Since the mutual fund industry is a device to keep the model tractable, the restriction in international insurance markets appears to be appropriate.

Workers. Individuals are born workers and start their life with zero assets. A generic worker i chooses consumption $C_t^w(i)$ and assets $A_{t+1}^w(i)$ to solve

$$V_t^w(i) = \max \left\{ (C_t^w(i))^\rho + \beta [\omega V_{t+1}^w(i) + (1 - \omega) V_{t+1}^r(i)]^\rho \right\}^{\frac{1}{\rho}}, \quad (8)$$

subject to

$$A_{t+1}^w(i) = R_{W,t} A_t^w(i) + W_t - T_t^w - C_t^w(i), \quad (9)$$

where W_t represents the market wage and T_t^w is the total amount of lump-sum taxes paid by a single worker. The function $V_{t+1}^r(i)$ (defined below) enters the continuation value of a worker to discount the possibility that retirement occurs between time t and $t + 1$.

Retirees. A retiree j chooses consumption $C_t^r(j)$ and assets $A_t^r(j)$ to solve

$$V_t^r(j) = \max \left\{ (C_t^r(j))^\rho + \beta \gamma_{t,t+1} [V_{t+1}^r(j)]^\rho \right\}^{\frac{1}{\rho}}, \quad (10)$$

subject to

$$A_{t+1}^r(j) = \frac{R_{W,t} A_t^r(j)}{\gamma_{t-1,t}} - C_t^r(j). \quad (11)$$

There is no public pension scheme and the only source of income for retirees is financial wealth.¹¹

Firms. Firms operate in perfect competition using a constant returns to scale technology with no adjustment costs. The aggregate production function is Cobb-Douglas

$$Y_t = (X_t N_t^w)^\alpha K_t^{1-\alpha}, \quad (12)$$

where $\alpha \in (0, 1)$ is the labor share and X_t stands for exogenous labor-augmenting productivity which grows at the constant rate x . In order to be consistent with the notation adopted so far, I assume that households own the capital stock but that firms bear the cost of depreciation δ .

Government. The government levies lump-sum taxes and issues one-period debt B_{t+1} to finance a given amount of spending G_t according to the flow budget constraint

$$B_{t+1} = R_{W,t} B_t + G_t - T_t, \quad (13)$$

¹¹Gertler [1999] studies the effects of moving from a public to private pension system in a closed economy with the life-cycle structure adopted in this model.

where $T_t = N_t^w T_t^w$ represents the total tax revenues.

The Foreign country has a completely symmetric structure.

4 Equilibrium in the World Economy

The environment is competitive and all agents take prices as given. Formally, a competitive equilibrium is a sequence of quantities and prices such that in each country (i) households maximize utility subject to their budget constraint, (ii) firms maximize profits subject to their technology constraint, (iii) the government chooses a path for taxes and debt, compatible with intertemporal solvency, to finance an exogenous level of total spending, (iv) goods and asset markets clear at the world level.

I can now describe in details the equations that constitute the competitive equilibrium.

Again, the focus is on the Home country. For simplicity, I start from the problem of a retiree. As demonstrated by Farmer [1990], under the assumption of risk neutrality in preferences [7], the solution of the individual problem consists of an explicit decision rule for consumption, given prices and policy variables.¹² A retiree consumes optimally a fraction out of total wealth according to

$$C_t^r(j) = \frac{\xi_t R_{W,t} A_t^r(j)}{\gamma_{t-1,t}}. \quad (14)$$

From the Euler equation, it is possible to show that the marginal propensity to consume evolves according to

$$\xi_t = 1 - \gamma_{t,t+1} \beta^\sigma R_{W,t+1}^{\sigma-1} \frac{\xi_t}{\xi_{t+1}}, \quad (15)$$

The crucial point of equation [15] is that the marginal propensity to consume for a retiree is independent of individual characteristics. Indeed, the evolution of ξ_t is governed only by the world interest rate (an aggregate variable) and the probability to survive (identical across retirees).

The solution for the problem of a worker can be characterized along similar lines. Workers consume out of financial and human wealth according to

$$C_t^w(i) = \pi_t [R_{W,t} A_t^w(i) + H_t^w(i)], \quad (16)$$

where π_t is the marginal propensity to consume for a worker. As for retirees, from the Euler

¹²The details of the derivation are provided in the appendix. See also Backus, Routledge and Zin [2004] for a discussion of other examples in the literature.

equation, it is possible to retrieve the law of motion of the workers' marginal propensity to consume

$$\pi_t = 1 - \beta^\sigma (\Omega_{t+1} R_{W,t+1})^{\sigma-1} \frac{\pi_t}{\pi_{t+1}}, \quad (17)$$

where $\Omega_t \equiv \omega + (1 - \omega) \epsilon_t^{\frac{1}{1-\sigma}}$ and $\epsilon_t \equiv \xi_t / \pi_t$ are two additional variables that link the marginal propensities to consume of the two cohorts.¹³ Also for workers, the marginal propensity to consume only depends on aggregate or cohort-specific variables but not on individual characteristics. The variable $H_t^w(i)$ in [16] represents the present discounted value of current and future human wealth net of taxation, that is

$$H_t^w(i) \equiv \sum_{v=0}^{\infty} \frac{W_{t+v} - T_{t+v}^w}{\prod_{s=1}^v (\Omega_{t+s} R_{W,t+s} / \omega)} = W_t - T_t^w + \frac{H_{t+1}^w(i)}{\Omega_{t+1} R_{W,t+1} / \omega}. \quad (18)$$

The term $\Omega_{t+1} R_{W,t+1} / \omega$ constitutes the effective discount rate for a worker. The first component of the higher discounting captures the effect of the finite lifetime horizon (less value attached to the future). The actual discount factor is also increased by the term ω which induces higher savings to finance consumption during the retirement period (positive probability of retiring). I now proceed to derive the aggregate consumption function.

The first step consists of aggregating consumption by cohort, which is possible because individuals within the same cohort share the same marginal propensity to consume. Aggregate variables take the form $Z_t^r \equiv \int_0^{N_t^r} Z_t^r(j) dj$ for retirees and $Z_t^w \equiv \int_0^{N_t^w} Z_t^w(i) di$ for workers.

Aggregate consumption for retirees is given by

$$C_t^r = \xi_t R_{W,t} A_t^r, \quad (19)$$

where A_t^r is total wealth that retirees carry from period $t - 1$ into period t , with aggregate return given by $R_{W,t}$.¹⁴

Individual consumption for workers can be aggregated along the same lines to yield

$$C_t^w = \pi_t (R_{W,t} A_t^w + H_t), \quad (20)$$

¹³In the appendix, I formally show that ϵ and Ω are both larger than one in steady state. For plausible parameter choices, including the calibration adopted in the numerical experiment section, this result holds also outside the steady state. The fact that $\epsilon_t > 1$ implies that the marginal propensity to consume is higher for retirees than for workers. As the next section clarifies, this result constitutes a key element to understand the response of consumption and savings to changes in demographic factors.

¹⁴Each individual retiree earns a return given by $R_{W,t} / \gamma_{t-1,t}$ but only a fraction $\gamma_{t-1,t}$ of retirees survive between two periods.

where A_t^w is total aggregate financial wealth held by workers. The aggregate value of human wealth evolves according to

$$H_t = N_t^w W_t - T_t + \frac{H_{t+1}}{(1 + n_{t,t+1}) \Omega_{t+1} R_{W,t+1} / \omega}. \quad (21)$$

The second step consists of writing the aggregate consumption function C_t as the sum of [19] and [20]. I denote by $\lambda_t \equiv A_t^r / A_t$ the share of total assets A_t held by retirees. It follows that the aggregate consumption function

$$C_t = \pi_t [(1 - \lambda_t) R_{W,t} A_t + H_t + \epsilon_t \lambda_t R_{W,t} A_t]. \quad (22)$$

Finally, I need to characterize the evolution of the distribution of assets, as captured by λ_t . This additional state variable keeps track of the heterogeneity in wealth accumulation introduced by the life-cycle hypothesis. Aggregate assets for retirees depend on the total savings of those who are retired in period t but also on the total savings of the fraction of workers who retire between t and $t + 1$

$$A_{t+1}^r = R_{W,t} A_t^r - C_t^r + (1 - \omega) (R_{W,t} A_t^w + N_t^w W_t - T_t - C_t^w). \quad (23)$$

Aggregate assets for workers depend only the savings of the fraction of workers who remain in the labor force

$$A_{t+1}^w = \omega (R_{W,t} A_t^w + N_t^w W_t - T_t - C_t^w). \quad (24)$$

After substituting expressions [19] and [24] into [23], I exploit the definition of λ_t to obtain the law of motion for the distribution of wealth across cohorts

$$\lambda_{t+1} A_{t+1} = (1 - \omega) A_{t+1} + \omega (1 - \xi_t) R_{W,t} \lambda_t A_t. \quad (25)$$

Expression [25] relates the evolution of the distribution of wealth λ_t to the aggregate asset position A_t .

The problem of the firm is standard. Firms hire labor to the point that the real wage equals the marginal product of labor

$$W_t = \alpha \frac{Y_t}{N_t^w}. \quad (26)$$

The investment decisions are consistent with the condition that the marginal product of capital, net of depreciation, equals the real return that prevails in the international asset

markets

$$R_{W,t} = (1 - \alpha) \frac{Y_t}{K_t} + (1 - \delta). \quad (27)$$

Finally, in equilibrium, the government is bound to choose combinations of taxes and debt that respect the intertemporal solvency condition

$$R_{W,t} B_t = \sum_{v=0}^{\infty} \frac{(T_{t+v} - G_{t+v})}{\prod_{s=1}^v R_{W,t+s}}. \quad (28)$$

The current value of debt (principal and interests) must be equal to the present discounted value of current and future primary surpluses. For later purposes, it is also convenient to express fiscal policy variables as fractions of GDP. Given the path for G_t , I assume that the government fixes the ratio $g_t \equiv G_t/Y_t$ and chooses combinations of tax rates $\tau_t \equiv T_t/Y_t$ and debt-to-GDP ratios $b_t \equiv B_t/Y_t$ that satisfy [28].

So far, I have described the behavior of the private sector and of the government of the Home country. The Foreign country is characterized by a similar set of relations. I now complete the characterization of the equilibrium by discussing how the asset and goods markets clear. The portfolio of assets held by the private sector is composed by capital, government bonds and net foreign assets

$$A_t = K_t + B_t + F_t. \quad (29)$$

The difference between production Y_t and domestic absorption ($C_t + I_t + G_t$) defines net export as

$$NX_t = Y_t - (C_t + I_t + G_t), \quad (30)$$

where investment is given by

$$K_{t+1} = I_t + (1 - \delta) K_t. \quad (31)$$

The evolution of net foreign assets links the goods and the asset markets together. Net foreign assets represent the payment received from the rest of the world in exchange for exporting

goods.¹⁵ Hence, F_t evolves according to

$$F_{t+1} = R_{W,t}F_t + NX_t. \quad (32)$$

International capital flows equalize the return $R_{W,t}$ across countries. The model is closed by the equation that characterizes the equilibrium in the market for international debt

$$F_t + F_t^* = 0.$$

Despite the tractability of the model, a complete closed form solution cannot be derived. In the appendix, I characterize a symmetric steady state of the model in which exogenous variables are constant and equal across countries. Given the presence of exogenous growth, quantities are not stationary. A steady state with balanced growth exists for detrended variables, that is, for variable expressed in terms of efficiency units $X_tN_t^w$. In such a steady state, quantities grow at the constant rate $(1+x)(1+n) \cong 1+x+n$. In what follows, I denote a generic variable Z_t in terms of efficiency units by lower case letters $z_t \equiv Z_t / (X_tN_t^w)$. The solution for the steady state and for the transition dynamics is obtained by employing a non-linear Newton method.¹⁶ In order to have a determinate steady state, I calibrate from the data an initial level of steady state government debt relative to GDP as the “fiscal rule” which closes the model.

As a final remark, it is interesting to note that in this model the steady state is determinate.¹⁷ The life-cycle structure pins down endogenously the steady state value of net foreign asset. Ghironi [2003] shows a similar result in a framework with overlapping families of infinitely lived agents based on Weil [1989].

¹⁵Following Obstfeld and Rogoff [1996], the current account balance can be defined as the one-period variation in the net foreign asset position

$$CA_t \equiv F_{t+1} - F_t = (R_{W,t} - 1)F_t + NX_t.$$

For the U.S., the current account and trade balance essentially coincide, which means that the net interest rate payment on the outstanding stock of net foreign assets must be small. Lane and Milesi-Ferretti [2001] and Gourinchas and Rey [2005], among others, discuss extensively these valuation effects.

¹⁶See Juillard [1996] for details about the implementation of the specific Newton-Raphson algorithm used to solve the model.

¹⁷It is well known that open economy models with incomplete markets generally feature steady state indeterminacy and non-stationary dynamics of net foreign assets. See Schmitt-Grohé and Uribe [2003] for the discussion of a number of alternative mechanisms to circumvent this problem.

5 The Determinants of the Trade Balance

The analytical tractability of the life-cycle framework presented in the previous two sections allows to isolate the determinants of the trade balance and to sort out precisely the equilibrium effect of demographic factors and fiscal policy on each component.

The assumption of perfect financial market integration implies that in each period capital in efficiency units must be equal across countries. The result follows from the no arbitrage relation

$$R_{W,t} - 1 + \delta = (1 - \alpha) (k_t)^{-\alpha} = (1 - \alpha) (k_t^*)^{-\alpha}, \quad (33)$$

which entails $k_t = k_t^*, \forall t$. Since both countries have access to the same technology, also output per efficiency unit is equalized across borders ($y_t = y_t^*$). Assuming that $g_t = g_t^*$, the national account identity [30] gives an expression for the trade balance as a function of consumption and investment differentials¹⁸

$$nx_t = -\frac{1}{2} (c_{R,t} + i_{R,t}), \quad (34)$$

where for any z_t and z_t^* , I define $z_{R,t} \equiv z_t - z_t^*$.

From [22] and its Foreign counterpart, relative consumption is

$$c_{R,t} = R_{W,t} \{[\pi_t (1 - \lambda_t) a_t - \pi_t^* (1 - \lambda_t^*) a_t^*] + (\xi_t \lambda_t a_t - \xi_t^* \lambda_t^* a_t^*)\} + (\pi_t h_t - \pi_t^* h_t^*) \quad (35)$$

The first two components of [35] represent the differentials in the propensity to consume out of financial wealth by workers and retirees respectively. The third term captures the differentials in the propensity to consume out of human wealth.

From [31] and its Foreign country counterpart, the equality of the capital stock in efficiency units implies that relative investments correspond to

$$i_{R,t} = (n_{t,t+1} - n_{t,t+1}^*) k_t. \quad (36)$$

I begin by highlighting the effects of population aging on the trade balance. For the sake of simplicity, it is useful to think of this experiment in isolation, holding constant population growth rates and government debt at their initial steady state values. In the absence of population growth rate differentials, $i_{R,t} = 0$. From [34], savings differentials completely

¹⁸In what follows, I refer to $c_{R,t}$ also as (the negative of) relative savings. The result is a consequence of the definition of aggregate savings as investment plus the trade rather than the current account balance.

characterize the dynamics of the trade balance

$$nx_t = -\frac{1}{2}c_{R,t}.$$

The adjustment of the trade balance to a change in the degree of population aging depends primarily on the response of the marginal propensities to consume. Different marginal propensities to consume within the same cohort across countries reflect different degrees of population aging between the Home and the Foreign economy.

In the model, the effects of population aging can be studied in relation to variations in the probabilities of survival γ and γ^* . In steady state, the relative marginal propensity to consume for retirees is

$$\xi_R = -\beta^\sigma R_W^{\sigma-1} (\gamma - \gamma^*).$$

Faster aging in one country is associated with a relatively more pronounced decrease in the marginal propensity to consume. While population aging also entails general equilibrium effects on the world interest rate, aging differentials only affect relative savings.

The effect of aging on the relative marginal propensity to consume for workers propagates through the terms Ω and Ω^*

$$\pi_R = -\beta^\sigma R_W^{\sigma-1} \left[(\Omega)^{\sigma-1} - (\Omega^*)^{\sigma-1} \right].$$

In the appendix, I show formally that, for a given interest rate, a higher surviving probability induces a reduction in the marginal propensity to consume for workers too. As before, different surviving probabilities across countries only influence relative savings.

Intuitively, an increase in the expected lifetime horizon brings about two effects. First, an individual who is in the labor force at the moment of the shock increases her savings to finance a longer retirement period. Second, after retirement, agents spread their consumption over an extended period. While it is still true that the marginal propensity to consume is higher for a retiree than for a worker, the consumption level of both cohorts decreases as compared to before the shock. Most importantly, in relative terms, the level of savings is higher in the country with higher life expectancy. The relative excess of savings generates a trade surplus associated with the accumulation of a positive stock of net foreign assets. The life-cycle structure of the population is at the core of this mechanism. The difference in the marginal propensities to consume is a result of the heterogeneity between workers and retirees. Despite the assumptions made to limit the degree of heterogeneity within each cohort and maintain analytical tractability, the model still preserves the key ingredients of

standard life-cycle frameworks and provides a transparent illustration of the forces at work.

Alongside the effects on the marginal propensity to consume, a higher probability of survival also increases the dependency ratio. From [6], in steady state, the cross-country differential is

$$\psi_R = \frac{(1 - \omega)(\gamma - \gamma^*)}{(1 + n - \gamma)(1 + n - \gamma^*)}.$$

Quite obviously, the dependency ratio is higher in the country with longer life expectancy. Since retirees have a higher marginal propensity to consume relative to workers, over time this effect partly offsets the initial fall of aggregate consumption. Nonetheless, for plausible parameterizations, the reduction of the marginal propensity to consume among retirees dominates the cross-sectional adjustment within each country.

Finally, the excess of total savings generates a drop in the interest rate. The absolute drop depends on the overall change in the lifetime horizon. In particular, the longer the retirement period, the larger the effect on workers and retirees savings. The effect of population aging appears to be crucial to explain the declining trend in the real interest rate during the last twenty five years.

I shall now discuss the impact of population growth rate differentials. As for the previous experiments, it is useful to fix the other sources of variations, in this case fiscal deficits and population aging, at their initial steady state values. In steady state, total population N_t grows at the same rate as the labor force N_t^w . During the transition, the link between the two growth rates in the Home country is governed by

$$\frac{N_{t+1}}{N_t} = \frac{(1 + n_{t,t+1}) + (1 - \omega) + \gamma\psi_t}{1 + \psi_t}.$$

The primitive shocks for this experiment are the labor force growth rates in the two countries (n and n^*).

The direct channel of transmission of population growth rate differentials operates through the investment sector. From equation [36], a smaller population growth rate in one region shifts relative investment towards the rest of the world. The no arbitrage relation [33] represents the driving force behind this adjustment. In frictionless international financial markets, the rate of return must be equal across countries. Capital flows towards the region with a relatively larger pool of workers ensure no arbitrage opportunities by employing capital where it is relatively more productive.¹⁹ If, for example, the Home country labor force grows

¹⁹The importance of this channel depends on the assumption about the degree of capital mobility across countries. For example, the presence of adjustment costs in investment would limit the transmission of population growth rates shocks.

at a relatively higher rate, Foreign capital relocates to the more productive Home firms, in exchange for a claim on ownership. This mechanism results in an accumulation of net foreign debt by Home households and a negative trade balance ($nx_t < 0$).

The presence of life-cycle behavior partly acts to counterbalance the investment channel. A smaller growth rate of the labor force at time t also implies an increase in the dependency ratio, which in turn leads to a reduction of the total population growth rate at $t+1$. Holding fixed the probability of survival, aggregate consumption increases because retirees become relatively more numerous. In the absence of population aging, the marginal propensities to consume for both cohorts are equal across countries. While the general equilibrium effect of a shock to population growth rates on the interest rate reduces the marginal propensities to consume for both cohorts, the marginal propensity to consume for retirees rises relative to workers.

The sign of the net adjustment (investment versus consumption-saving decisions) is ambiguous. The numerical experiments in the next section suggest a mild predominance of the investment channel but the overall effect appears to be rather small.

Lastly, I consider the effects of fiscal deficits on the trade balance. In what follows, I shut off any demographic differential across countries. For the sake of the argument, it is also convenient to set the population growth rate equal to zero.²⁰ In such circumstances, the trade balance is again completely determined by relative savings

$$nx_t = -\frac{1}{2}c_{R,t}.$$

The absence of life expectancy differentials implies that the marginal propensity to consume for each cohort is identical across countries. Expression [35] can be rewritten as

$$c_{R,t} = \pi_t [h_{R,t} + R_{W,t}a_{R,t} + (\epsilon_t - 1) R_{W,t} (\lambda_t a_t - \lambda_t^* a_t^*)]. \quad (37)$$

The first component of [37] is relative human wealth, which depends negatively on the relative tax rate

$$h_{R,t} = - \sum_{v=0}^{\infty} \frac{\tau_{R,t+v} y_{t+v}}{\prod_{s=1}^v [\Omega_{t+s} R_{W,t+s} / (1+x)\omega]}. \quad (38)$$

The second term of [37] is the gross return on the relative aggregate asset position. From

²⁰This assumption permits to illustrate more starkly the non-Ricardian effects associated to fiscal policy in this framework. Weil [1989] shows that a positive population growth rate (overlapping families of infinitely lived agents) is a sufficient condition for Ricardian equivalence to fail in an otherwise neoclassical world.

expression [28] and [29], I can rewrite

$$R_{W,t}a_{R,t} = 2R_{W,t}f_t + \sum_{v=0}^{\infty} \frac{\tau_{R,t+v}y_{t+v}}{\prod_{s=1}^v [R_{W,t+s}/(1+x)]}. \quad (39)$$

Equation [39] follows from the fact that capital in efficiency units is equal across countries and that the Home and Foreign net foreign asset positions add up to zero. Finally, the third element of [37] captures relative asset holdings by retirees (remember that $\lambda_t = A_t^r/A_t$). From expression [25], this difference evolves according to

$$\lambda_t a_t - \lambda_t^* a_t^* = (1 - \omega) a_{R,t} + \omega \left(\frac{1 - \xi_t}{1 + x} \right) R_{W,t-1} (\lambda_{t-1} a_{t-1} - \lambda_{t-1}^* a_{t-1}^*). \quad (40)$$

In general, relative asset holdings by retirees can be expressed as a function of the cross-country difference in aggregate assets.²¹

The solution for the trade balance in the absence of cross-country demographic differentials clarifies the channel through which fiscal deficits influence the trade balance. Starting from a symmetric steady state, consider, for example, the case of a relative fiscal deficit in the Home country. This experiment corresponds to an increase of the debt-to-GDP ratio b_t holding constant the level of spending, so that the tax rate τ_t is reduced during the expansion. In this scenario, on impact, the trade balance depends only on future relative tax rates. Since the Home country is running a fiscal deficit, the relative tax rate is negative, boosting relative human wealth and turning the trade balance into a deficit. As time goes by, the tax rate in the Home country increases in order to smooth the tax hike that takes place at the end of the fiscal expansion. Along the transition, the tax cut stimulates private consumption and reduces private savings. In general equilibrium, the drop of the savings rate increases the real interest rate. This effect partially limits the benefits of the fiscal expansion on consumption.

Fiscal deficits entail non-Ricardian effects because of the life-cycle structure of the economy. In expressions [38] and [39], relative taxes are discounted at different rates. In other words, a fraction of government debt constitutes net wealth for the private sector. Because of transition into retirement, a worker who benefits from a tax cut today, with some probability, will not be subject to the future higher tax rate, which is necessary, *ceteris paribus*, to satisfy the

²¹Whether expression [40] admits a backward or forward solution depends on the autoregressive coefficient being larger or smaller than one. Fiscal and demographic forces will typically exert opposite effects on this term. For high enough values of the equilibrium interest rate, possibly due to expansionary fiscal policies, the autoregressive coefficient will be larger than one so that only current and future fiscal decisions matter for the equilibrium trade balance.

government intertemporal solvency condition. Hence, the capitalization of future tax rates by workers is less than complete (as measured by the factor Ω_{t+v}/ω). In this case, fiscal and trade deficits are “twins”, although the synchronization is less than perfect. The persistence in the trade deficit is augmented by the term in [40], which measures the variation in the distribution of wealth across cohorts following an expansion and is peculiar of the life-cycle structure of the economy.

As a final remark, when $\omega = 1$, the setup encompasses the standard infinite horizon model with a representative agent. Under this limiting case, agents never retire so that $\lambda_t = \lambda_t^* = 0$. Since also $\Omega_t = 1 \forall t$, Ricardian equivalence holds and the trade balance is only a function of the (relative) level of public spending, not of the financing decisions (debt versus taxes).

6 A Quantitative Investigation of the U.S. Trade Deficit

In this section, I evaluate the quantitative significance of the different effects discussed above. The objective is to assess the overall contribution and the relative importance of demographic factors and fiscal deficits in accounting for the evolution of the U.S. trade balance vis-a-vis the G6 after the breakdown of Bretton Woods. The application of the theoretical apparatus developed so far leads to three main results:

1. The demographic differentials between the U.S. and the G6 explain the declining trend of the U.S. trade deficit in the last three decades.
2. The consideration of demographic factors reconciles the evolution of fiscal policy in the U.S. and in the G6 with the dynamics of the trade balance.
3. The demographic transition generates a path of the world real interest rate broadly consistent with the data.

The time period is one year. The demographic transition starts in 1970 and convergence for population growth rates²² occurs in 2030. The choice of 1970 as the base year depends upon two facts. First, the assumption of perfect capital markets appears to be the most appropriate for the period after the collapse of Bretton Woods in the early 1970s. Second, in those same years, demographic differentials between the U.S. and the G6 were also minimal. It seems then reasonable to start from a symmetric steady state in which trade is balanced and

²²In order to avoid that, in the limit, one country ends up representing the entire world economy, I consider convergence to a steady state in which both regions grow at the same rate.

evaluate the impact of fiscal deficits and demographics without adjustment effects induced by an existing stock of assets or liabilities. In the section on robustness, I discuss alternative initial steady states as well as different convergence scenarios.

Initial Steady State. The calibration of the demographic variables is based on previous studies and on data from the United Nations World Population Prospects: The 2004 Revision. Population grows at a rate of 1%, which is in line with the observed value for both regions in 1970. Individuals are assumed to enter the economy as workers at the age of 20 and work on average $(1 - \omega)^{-1}$ years. I choose the parameter $\omega = 0.9778$ to match a 45-year average permanence in the labor force, which corresponds to the working period used by Auerbach and Kotlikoff [1987]. I set the surviving probability for a retiree (γ) equal to 0.8, in order to match the average expected lifetime horizon of an agent in the model ($65 + (1 - \gamma)^{-1}$) to the empirical counterpart in 1970, which was roughly 70 years both for the U.S. and G6.

The choice of the values for preference and production parameters is mostly in line with the existing literature. The elasticity of intertemporal substitution σ is assumed to be equal to 0.5, which represents a compromise between the values usually adopted in the public finance literature (e.g. Auerbach and Kotlikoff [1987]) and in the real business cycle literature (e.g. Cooley [1995]).²³ The discount factor β is set residually equal to one²⁴ in order to obtain a real interest rate of 5% in 1970. The production parameters take values in line with the real business cycle literature. The labor share α is assumed to be 2/3, the depreciation rate δ is set to 6% and technology grows at 1%.

Throughout the analysis, I focus on the government financing decisions of a constant level of public expenditure. I fix the ratio of government spending to GDP to 20%, which corresponds approximately to the average U.S. value during the post-war period,²⁵ as reported by the Bureau of Economic Analysis. The ratio of government debt to GDP is calibrated using data on net debt from the International Monetary Fund (World Economic Outlook Database), which are available only starting in 1980. I set the ratio of debt to GDP in the initial symmetric steady state to 26%, which represents the average between U.S. and G6 in 1980. For the U.S., this value represents a reasonable approximation also for the 1970s, a decade

²³See Attanasio [1999] for a comprehensive survey.

²⁴A value of the subjective discount factor equal to one is consistent with the estimates in Hurd [1989].

²⁵This value obviously underestimates the role of government spending in European countries and Japan. However, the model excludes features, such as a public pension system or a public health system, which account for large parts of government spending in those countries and are likely to provide utility to households. Hence, the value of 20% seems a reasonable approximation in a context in which government spending is considered pure waste.

during which the ratio of net debt to GDP was roughly constant.²⁶ In the baseline experiment, I assume that the ratio of debt to GDP is constant at 26% also in the G6 throughout the 1970s. In the section on robustness, I consider an extended time series for the G6 extrapolated from gross government debt as reported by the OECD Economic Outlook Database.

Table 1. INITIAL SYMMETRIC STEADY STATE

n	1%	POPULATION GROWTH RATE
$(1 - \omega)^{-1}$	45	AVG. WORKING PERIOD (YEARS)
$(1 - \gamma)^{-1}$	5	AVG. RETIREMENT PERIOD (YEARS)
β	1	SUBJECTIVE DISCOUNT FACTOR
σ	0.5	ELASTICITY OF INTERTEMPORAL SUBSTITUTION
α	2/3	LABOR SHARE
δ	6%	DEPRECIATION RATE
x	1%	GROWTH RATE OF TECHNOLOGY
g	20%	GOVERNMENT SPENDING (% OF GDP)
b	26%	GOVERNMENT DEBT (% OF GDP)

Given the benchmark calibration, the model delivers a capital-income ratio equal to 3 in the initial steady state. Investment constitutes 24% of GDP and the tax rate is equal to 20.8%.

Final Steady State. The transition from the initial to the final steady state is governed by the evolution of demographic factors (expected lifetime horizons and population growth rates) and of fiscal stances (debt-to-GDP ratios) in both countries. Convergence for population growth rates occurs in 2030 at a value equal to 0.2%, in the baseline experiment, which coincides with the average of the U.N. projections for the U.S. and the G6 for the period 2030 – 2050. The U.N. projections also constitute the anchor for the lifetime expectancy in both regions in the same period. The implied values of the average retirement period are 15 and 18 years for the U.S. and G6 respectively. For debt-to-GDP ratios, I use the projected values for 2006 from the IMF World Economic Outlook Database, approximately equal to 50% in the U.S. and 70% in the G6.

²⁶Data on net debt for the U.S. are available also from the Congressional Budget Office for the entire post war period. While the pattern is essentially the same, there exist some differences in terms of absolute magnitudes, although the order never exceeds 5%. For international comparisons, the World Economic Outlook Database appears to be the most reliable source for the time series of net debt.

Table 2. FINAL STEADY STATE

	U.S.	G6	
n	0.2%	0.2%	POPULATION GROWTH RATE
$(1 - \gamma)^{-1}$	15	18	AVG. RETIREMENT PERIOD (YEARS)
b	50%	70%	GOVERNMENT DEBT (% OF GDP)

The Experiment. The central numerical experiment of the paper consists of simulating an unexpected shock to demographic variables and fiscal stances in 1970 and to compute the transition path to the final steady state, which is reached in 2030. Under perfect foresight, agents are surprised by the shock in the initial period but perfectly anticipate the evolution of the exogenous variables thereafter. I discuss the likely implications of this assumption below.

I use the realized values of net debt as a percentage of GDP in the two countries as the exogenous fiscal stances in the simulation. Given the constant level of government spending as a fraction of output, the tax rate and the government surplus are an outcome of the model which depends on the equilibrium output and interest rate. I compute the probability of surviving from the data on life expectancy under the assumption that the average permanence in the labor force is constant at 45 years. Finally, I infer the growth rate of the labor force from the data on population growth rates, adjusting for convergence in 2030 at 0.2%.

Results. Figure 5 compares the series of the trade balance as percentage of GDP generated by the model with the trend of the HP-filtered series of the U.S. trade balance vis-a-vis the G6 for the period 1980 – 2004.

The time series of the trade balance exhibits three key periods. The first segment, which includes the first “dip” in the trend, begins in the early 1980s and lasts for slightly less than a decade. The second part of the series covers the years from the late 1980s to the mid 1990s and is characterized by a small reduction in the trade deficit. Finally, the third period, which approximately starts in the second part of the 1990s and continues as of today, displays a further worsening of the trade deficit towards 2% of GDP.

The model performs reasonably well to explain the long run movements of the U.S. trade balance vis-a-vis the G6. It captures the overall declining trend, the prolonged period of sustained deficits and the key swings in the last twenty five years. The fraction of the trend explained by the model reduces over time but this aspect is most likely associated with the assumption of perfect foresight which leads to a more pronounced adjustment at the beginning of the experiment.

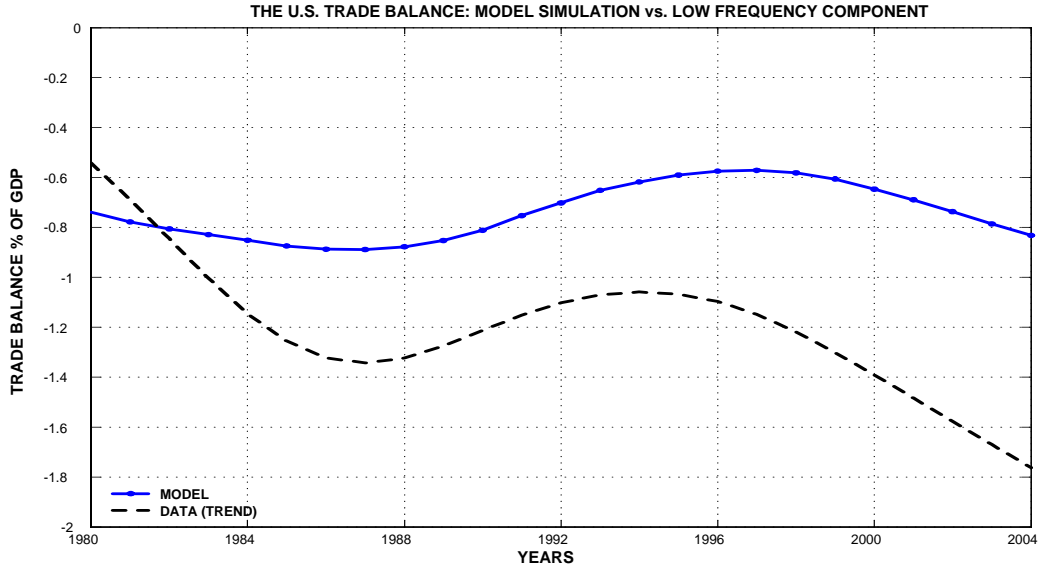


Figure 5: THE U.S. TRADE BALANCE - SIMULATION AND DATA (TREND).

The central argument of the paper revolves around the decomposition of the simulated trade balance series, reported in Figure 6. I repeatedly shut off all but one source of variation and compute the trade balance implied by the model under the a single shock. In what follows, I discuss the two main results of this exercise and I complete the analysis with the general equilibrium implications of the demographic transition.

Result #1. *Demographic differentials explain the negative trend of the U.S. trade balance.*

The demographic transition generates the negative trend of the trade balance observed in the data. As illustrated by Figure 6, the differentials in life expectancy account for most of the adjustment.

The intuition behind these effects hinges upon the discussion of the previous section. The response to the observed demographic transition requires the Home country to run a persistent trade deficit that accommodates the excess of savings flowing in from the rest of the world. While aging and smaller growth are a global phenomenon, the cross-country differentials are the key determinant of the equilibrium trade balance. In the Foreign country, workers save relatively more to cope with a longer retirement period and retirees spread their consumption over a relatively longer horizon.

The simulation also shows that the strength of the investment channel by itself is quantitatively limited. On the one hand, capital flows towards the Home country where it is

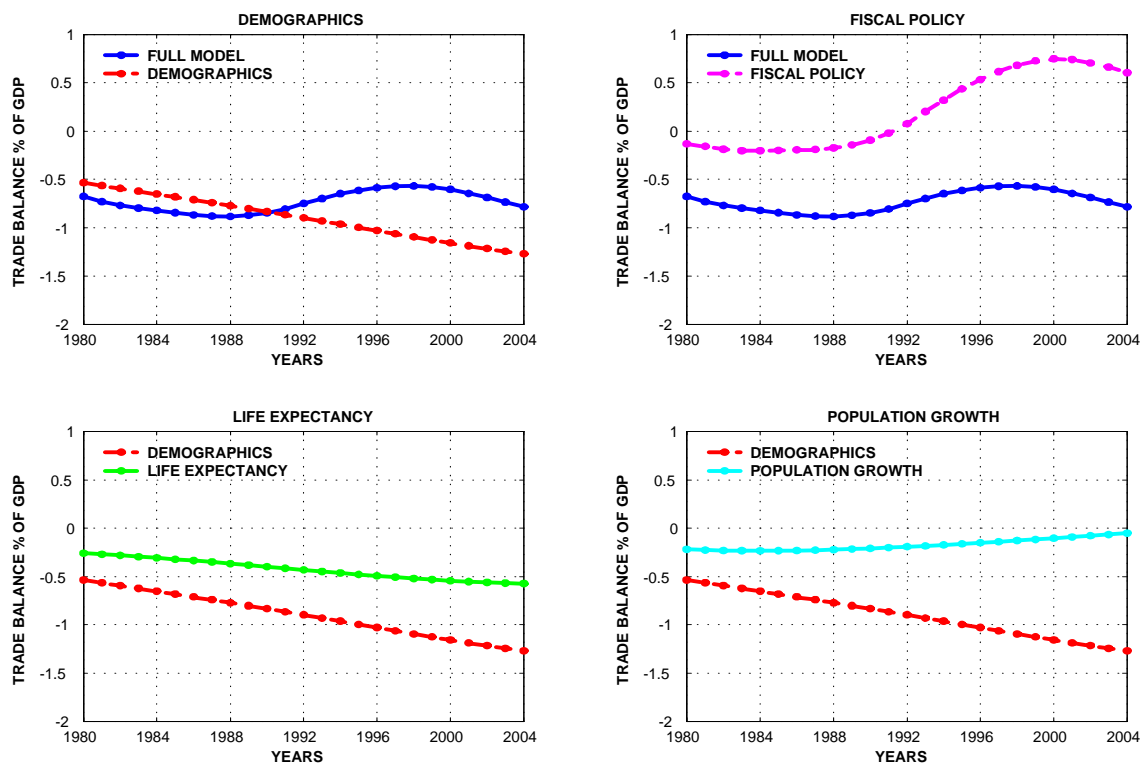


Figure 6: THE DETERMINANTS OF THE TRADE BALANCE.

relatively more productive. On the other hand, however, holding constant life expectancy, smaller population growth rates reduce savings in the Foreign country.

In the case of variable surviving probability, a smaller population growth rate interacts with population aging to the extent that both effects push the dependency ratio in the same direction. The number of retirees grows relatively to the number of workers because (i) the average lifetime horizon is longer (aging) and (ii) less people are born in each period (growth). The combination of these two forces leads to a drop in the marginal propensities to consume which is stronger for the economy experiencing a more dramatic demographic transition.

Overall, the negative trend in the Home country trade balance is the result of demographic differentials that persist over time.

Result #2. *Fiscal policy amplifies the trend induced by demographic differentials.*

The second notable feature of the quantitative experiment is that fiscal deficits operate as an amplification mechanism on top of the trend generated by demographic factors.

Fiscal policy differentials cause the three major swings in the simulated trade balance which replicate quite closely the fluctuations in the observed trend during the last twenty five years.

It is important to stress that fiscal policy differentials, rather than absolute magnitudes, are the ultimate determinant of the equilibrium trade balance. In a two-country world, fiscal deficits in the Home country do not necessarily entail a Home country trade deficit. Absent demographic differentials, a fiscal deficit could be accompanied by a trade surplus simply because fiscal policy in the Foreign country is relatively more expansionary. This consideration bears the consequence that fiscal policy alone can hardly be the main driving force behind the realized U.S. trade deficit vis-a-vis the G6. Differential fiscal stances in the data are simply too small to support large trade deficits, as already pointed out in section 2.

The bottom line of the analysis is that it is possible to reconcile the persistent U.S. trade deficits with the conduct of fiscal policy in the last twenty five years once demographic dynamics are properly taken into account. In this scenario, the overall picture of the U.S. trade balance vis-a-vis the G6 becomes fairly reasonable.

Result #3. *The behavior of the real interest rate is broadly consistent with the data.*

The demographic transition and the fiscal policy decisions in both regions carry important implications for the equilibrium world interest rate.

Figure 7 compares the world interest rate generated by the model as an outcome of the simulation discussed above with the data on the ex-post real interest rate for the G7 (from the IMF International Financial Statistics Database).

Contrary to the trade balance, the real interest rate is determined by global variables. The first lesson from Figure 7 is that demographic factors also play a critical role in accounting for the declining trend in the real interest rate. The excess savings associated with the increase in life expectancy at the world level drives the real interest rate downward. Second, fiscal deficits lead to an increase of the interest rate but the quantitative contribution is indeed modest.

These results shed new light on the empirical (lack of) relationship between fiscal deficits and interest rates. Evans [1987] suggests that the absence of high interest rates in periods of substantial fiscal deficits, both for the U.S. and at the international level, supports the hypothesis of Ricardian equivalence.²⁷ The analysis in this paper implies that, even if fiscal deficits do trigger a positive response of the real interest rate, the failure to control for demographic trends might substantially bias the results.

²⁷See also Elmendorf and Mankiw [1999] for a critical review of the literature on this theme.

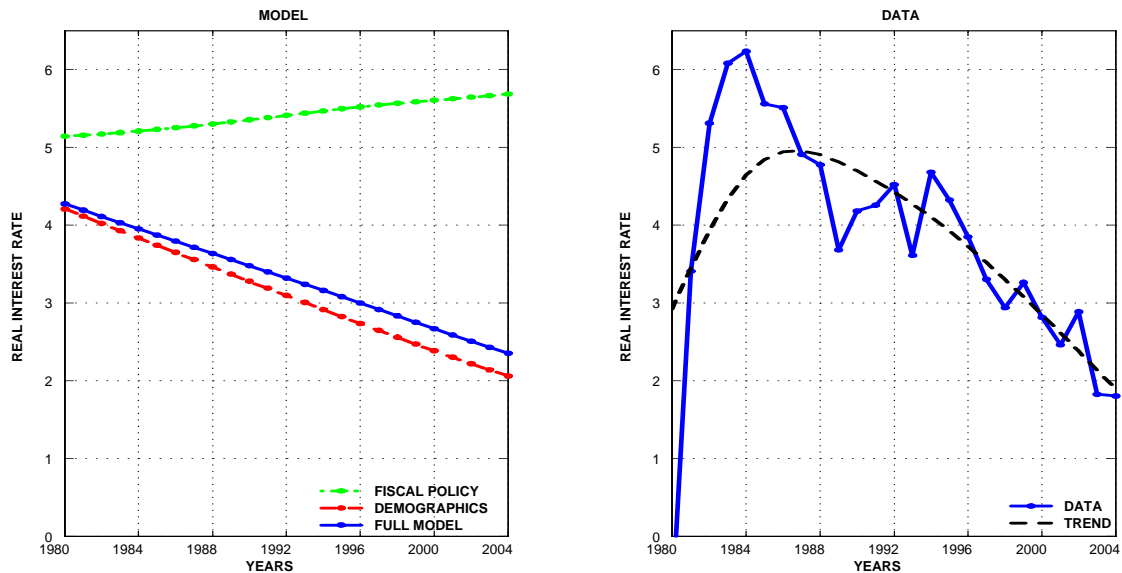


Figure 7: THE REAL INTEREST RATE - SIMULATION AND DATA.

Decreasing real interest rates generally favor deficits by reducing the cost of borrowing and maintaining a low burden of outstanding debt. This mechanism constitutes an intertemporal valuation effect for both net foreign debt and government debt which increases the persistence of existing deficits.

Figure 8 illustrates the valuation mechanism associated with low interest rates with two simple examples.

The left panel reports the response of the trade balance to an increase in life expectancy in the case of a two-country world and in the case of a small open economy which faces a constant interest rate. In the two-country model, the demographic shock hits the Foreign economy (γ^* increases from 0.8 to 0.85 over 10 years) and generates a trade deficit in the Home country according to the mechanism discussed in the previous sections. For a small open economy, the same demographic transition generates a trade deficit in the rest of the world which, in this case, represents the Home country. The simulation shows that the general equilibrium effects associated with the interest rate absorb on impact half of the adjustment in the trade balance. Moreover, a decreasing interest rate implies that the trade balance is more persistent and that the rebalancing towards the new steady state occurs more gradually

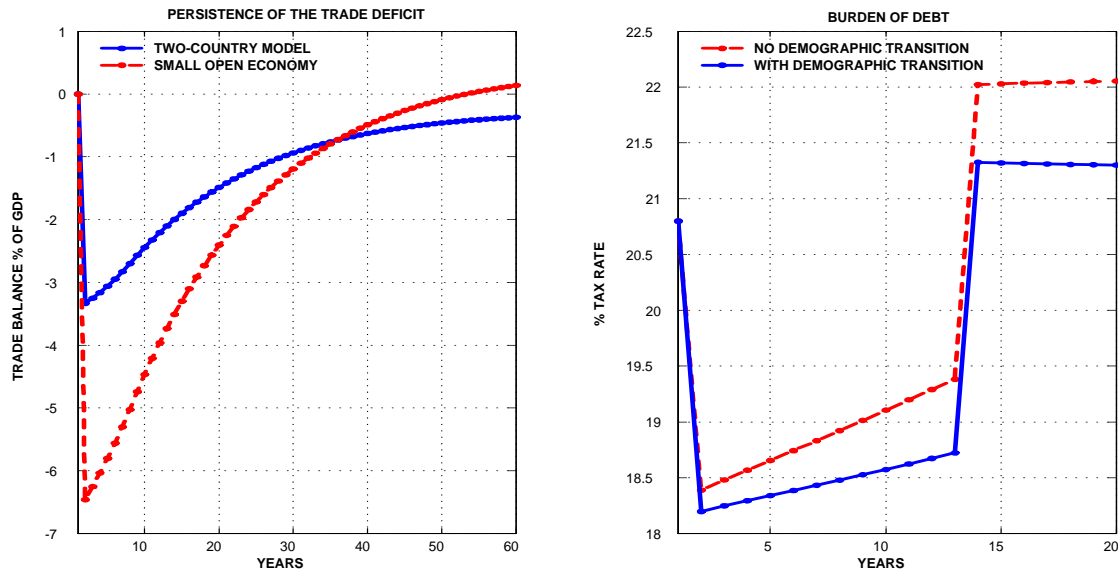


Figure 8: GENERAL EQUILIBRIUM EFFECTS.

given the reduced burden of foreign debt.²⁸

The right panel of Figure 8 displays the adjustment of the domestic tax rate after a permanent fiscal expansion in the Home country (b increases from 26% to 60% over 12 years) with and without demographic transition. The demographic transition consists of an increase in the probability of surviving γ from 0.8 to 0.85 over 10 years. The low interest rate associated with the demographic transition reduces the burden of outstanding public debt. Therefore, during the expansion, the government of the Home country can afford a larger reduction of the tax rate. In the new steady state, the tax rate for an economy that experiences a demographic transition is 1% lower than in the case of stationary demographics. This effect is indeed quantitatively relevant if one considers that the demographic transition occurs over a ten-year horizon and only leads to an increase of 1.5 years in life expectancy.

²⁸In this experiment, the unilateral demographic transition leads the Home country to run a long run trade surplus that compensates the accumulation of net foreign debt experienced along the transition.

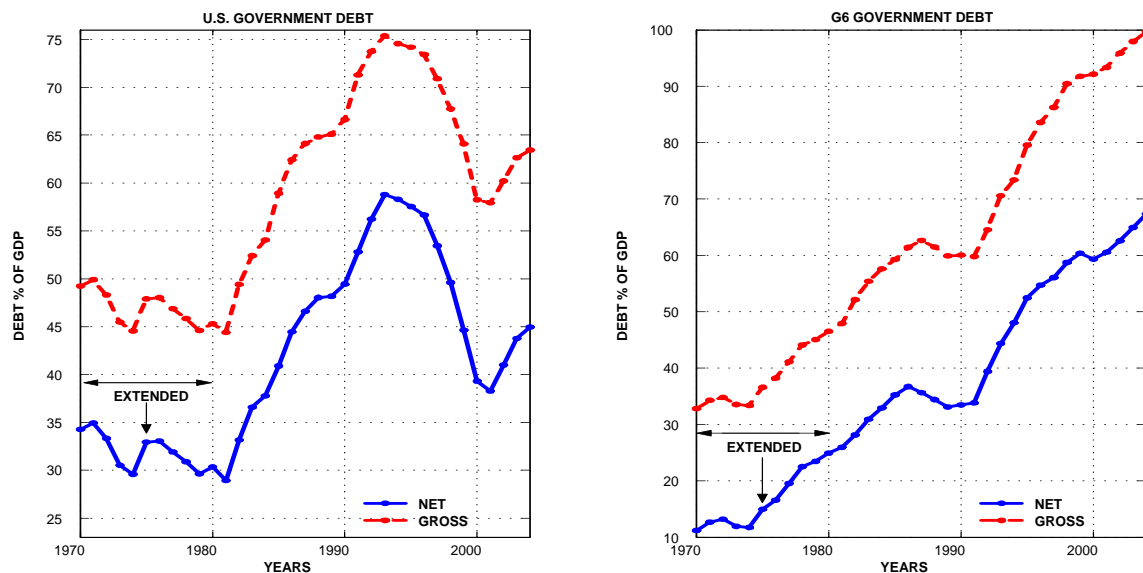


Figure 9: EXTENDED SERIES FOR NET GOVERNMENT DEBT.

6.1 Alternative Scenarios

The choice of the parameter values for the numerical experiments is fairly standard.²⁹ In this section, I discuss the sensitivity of the quantitative exercise presented above to alternative assumptions about the exogenous processes and the convergence scenarios.

In Figure 9, I extend the series for net government debt back to 1970 using data on gross government debt from the Organization for Economic Cooperation and Development (Economic Outlook Database). By itself, gross debt represents a less appropriate fiscal indicator than net debt for it includes holdings of government bonds by various branches of the public administration and hence overestimates the impact of public debt on private sector decisions. The inference of net debt from the gross stance conveys some degree of uncertainty associated with different national accounting standards and institutional frameworks, especially in G6 countries. Finally, for some countries such as France, Italy and U.K., data availability at the beginning of the sample is an issue also for the gross debt series, posing some additional problems in terms of extrapolation.

²⁹Gertler [1999] analyzes the sensitivity of the steady state interest rate and capital stock to alternative values for the elasticity of intertemporal substitution and finds a negative relationship between such parameter and the steady state interest rate. A low value of this elasticity also reinforces the crowding out effect of fiscal policy on the capital stock.

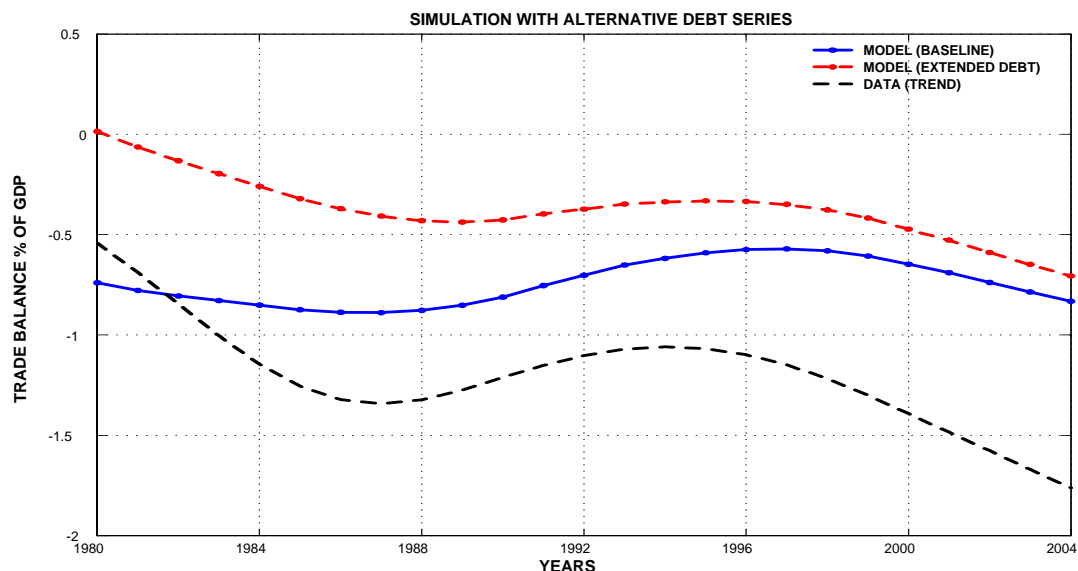


Figure 10: ALTERNATIVE SCENARIOS - DEBT SERIES.

Despite these limitations in terms of the data, the simulation under the extended government debt series adds an interesting element to the analysis to the extent that it permits to illustrate the effects of a non-symmetric initial steady state.

Figure 10 displays the simulated trade balance (in percentage of GDP) for the baseline scenario and for the case of the extended debt series. If the stock of debt is different across countries in the initial steady state, the country with higher debt experiences a trade surplus.

The steady state effect of debt differentials corresponds to a shift in the initial condition for the dynamics of net foreign debt. As the final steady state coincide with the benchmark experiment, the initial difference vanishes as time goes by. Qualitatively, the dynamic evolution of the trade balance appears to be unaffected. The model still generates a downward trend in the trade balance broadly consistent with the data, although of reduced magnitudes. Interestingly, the relative U.S. fiscal consolidation during the 1990s is now associated with a smaller correction in the trade deficits, due to the faster adjustment in relative savings.

Finally, while the steady state relative debt differs across countries, the total stock is basically unchanged with respect to the baseline scenario. Hence, the behavior of the equilibrium interest rate bears no consequences on the different evolution of the trade balance.

Table 3. CONVERGENCE SCENARIOS

POPULATION GROWTH RATE		LIFE EXPECTANCY	
		U.S.	G6
HIGH	0.3%	83	87
MEDIUM	0.2%	80	83
LOW	0.1%	78	80

A second aspect that requires some additional considerations concerns the values chosen for population growth rates and population aging in the final steady state. In the baseline experiment, the economy reaches the final steady state in 2030 with a population growth rate equal to 0.2%, which represents the average of the U.S. and G6 projected growth rates for the period 2030–2050. The U.N. projections however display significant cross-country differences for several additional years. Under the current estimates, convergence is predicted to occur only in 2200. The purpose of this part is to evaluate the effects of different assumptions about convergence. To be specific, besides the baseline (medium) scenario, I also consider the possibility that convergence occurs at higher (0.3%) or lower (0.1%) population growth rates. Similarly, I evaluate the effects of a final steady state in which population aging is less (more) pronounced and the two regions display smaller (larger) life expectancy differentials (78 – 80 and 83 – 87 for the U.S. and the G6 respectively). Table 3 summarizes the possible combinations. The benchmark experiment of the previous section is based on the “medium” scenario.

The results of the robustness analysis are reported in Figure 11. The left panel shows the trade balance under alternative assumptions for the population growth rate in the new steady state. The right panel displays the results in the case of different scenarios for life expectancy.

Alternative values of population growth rates shift the trade balance series roughly by the size of the difference (0.1%). The reason is that, in the full demographic transition, differentials in population growth rates operate mostly in the investment sector through contemporaneous flows of capital towards the region that grows at a faster rate.

The relationship between different patterns of population aging and the adjustment of the trade balance contains two main points. First, if life expectancy is higher in the new steady state, the adjustment of the trade balance is more pronounced. This result is fairly obvious and follows naturally from the analysis in the previous sections. Second, since the case of long life expectancy also involves larger steady state differentials, the trend component induced by

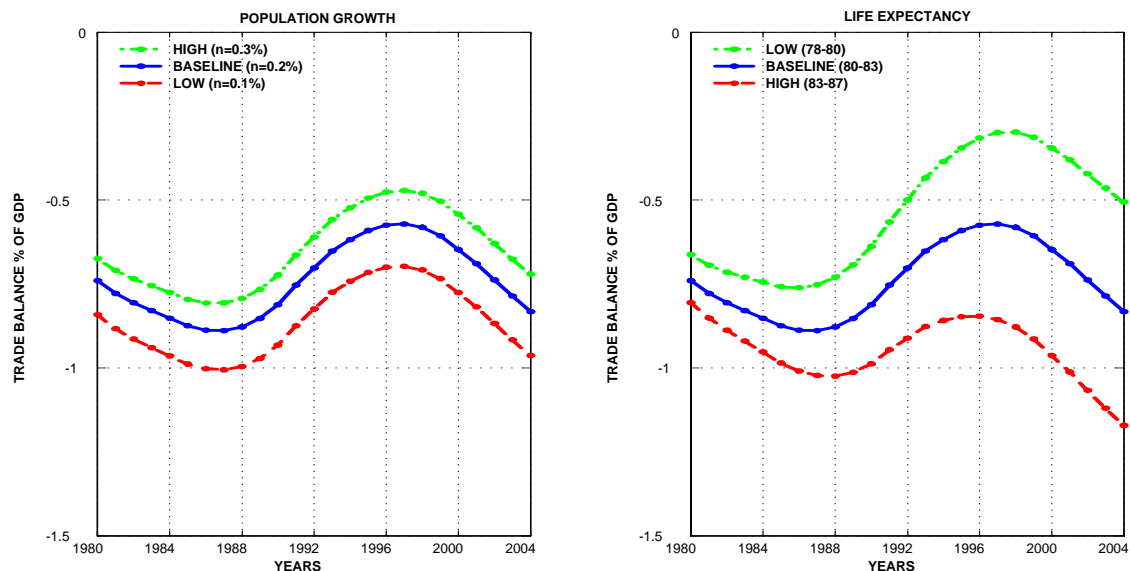


Figure 11: ALTERNATIVE SCENARIOS - DEMOGRAPHIC VARIABLES.

demographic factors dominates even more clearly over the short run fluctuations generated by fiscal policy.

I conclude this section with a final comment on the assumption of perfect foresight. In this paper, following the typical approach adopted in large scale OLG models, I have considered no aggregate uncertainty. While U.N. projections for the demographic transition are available until 2050, it seems unlikely that agents completely discount the exact aging profile and population growth rates in their saving and investment decisions. Perhaps even more importantly, the quantitative importance of fiscal deficits in determining the trade balance might change if aggregate uncertainty about government policy is considered. In the context of this work, I have decided to preserve the analytical tractability of the framework, perhaps at the cost of sacrificing the quantitative accuracy of some experiments. Given that agents in this economy perfectly anticipate the future movements of aggregate exogenous variables, it seems safe to anticipate that the inclusion of aggregate uncertainty would lead to less smoothing and more pronounced movements in the trade balance due to unexpected innovations.

7 Conclusions

This paper has stressed the importance of demographic factors to explain the behavior of the U.S. trade balance vis-a-vis the G6 in the last three decades.

The main idea contends that demographic differentials explain the negative and declining trend in the U.S. external position. In the context of a tractable life-cycle model, I have illustrated analytically and quantitatively how relative savings adjust in response to expectations of longer lifetime horizons. Saving differentials represent the driving force in accounting for capital flows from the G6 towards the U.S.

The second result of the paper is that, in an environment in which fiscal policy entails non-Ricardian effects, the demographic transition limits the rebalancing impact of fiscal consolidations and amplifies the expansionary consequences of fiscal deficits. The theoretical analysis demonstrates that the cross-country differentials of fiscal stances, rather than their absolute values, constitute the main determinant of trade imbalances. The quantitative experiments support the heuristic evidence that the fiscal differentials in the data are indeed too small to generate large movements in the U.S. balance of trade. Nonetheless, fiscal deficits appear to be associated with the major swings in the long run trend of the trade balance.

Finally, the interest rate generated by the model mimics well the declining trend observed in the data. The global excess of savings due to longer retirement periods leads to decreasing returns in financial markets which contribute to increase the persistence of deficits by reducing the burden of debt and lowering the cost of borrowing.

The analytical tractability of the model should prove useful to investigate a number of problems. For example, the heterogeneity of the population might deliver interesting insights for the study of optimal fiscal policy with distortionary tax instruments. Another possible application could analyze the welfare costs of fiscal rules, in relation to the current debate in the European Monetary Union. These questions are left for future research.

References

- [1] Attanasio, O. (1999), “Consumption”, in *Handbook of Macroeconomics*, J. Taylor and M. Woodford (eds.), North Holland – Amsterdam.
- [2] Attanasio, O., Kitao, S. and G. Violante (2005), “Global Demographic Trends and Social Security Reform”, *Working Paper*, University College London and New York University.
- [3] Auerbach, A. and L. Kotlikoff (1987), *Dynamic Fiscal Policy*, Cambridge University Press – Cambridge, UK.
- [4] Backus, D., Routledge, B. and S. Zin (2004), “Exotic Preferences for Macroeconomists”, in *National Bureau of Economic Research Macroeconomics Annual*, M. Gertler and K. Rogoff (eds.), The MIT Press – Cambridge, MA.
- [5] Bernanke, B. (2005), “The Global Saving Glut and the U.S. Current Account Deficit”, *Sandridge Lecture*, Virginia Association of Economics – Richmond, VA.
- [6] Blanchard, O. (1985), “Debt, Deficits and Finite Horizons”, *Journal of Political Economy* 93, pp. 223-247.
- [7] Bohn, H. (2004), “The Sustainability of Fiscal Policy in the United States”, *Working Paper*, University of California at Santa Barbara.
- [8] Brooks, R. (2003), “Population Aging and Capital Flows in a Parallel Universe”, *IMF Staff Papers* 50, pp. 200-221.
- [9] Caballero, R., Fahri, E. and P.O. Gourinchas (2005), “An Equilibrium Model of “Global Imbalances” and Low Interest Rates”, *Working Paper*, MIT and University of California at Berkeley.
- [10] Cooley, T. (1995), *Frontiers of Business Cycle Research*, Princeton University Press – Princeton, NJ.
- [11] Domeij, D. and M. Flodén (2004), “Population Ageing and International Capital Flows”, *Centre for Economic Policy Research Working Paper #4644*.
- [12] Elmendorf, D. and G. Mankiw (1999), “Government Debt”, in *Handbook of Macroeconomics*, J. Taylor and M. Woodford (eds.), North Holland – Amsterdam.

- [13] Engel, C. and J. Rogers (2005), “The U.S. Current Account Deficit and the Expected Share of World Output”, *Working Paper*, University of Wisconsin at Madison and Board of Governors of the Federal Reserve System.
- [14] Epstein, L. and S. Zin (1989), “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework”, *Econometrica* 57, pp. 937-969
- [15] Erceg, C., Guerrieri, L. and C. Gust (2005), “Expansionary Fiscal Shocks and the Trade Deficit”, *International Finance Discussion Paper #825*, Board of Governors of the Federal Reserve System.
- [16] Evans, P. (1987), “Do Budget Deficits Raise Nominal Interest Rates? Evidence from Six Countries”, *Journal of Monetary Economics* 20, pp. 281-300.
- [17] Farmer, R. (1990), “Rince Preferences”, *Quarterly Journal of Economics* 105, pp. 43-60.
- [18] Feroli, M. (2003), “Capital Flows Among the G-7 Nations: A Demographic Perspective”, *Finance and Economics Discussion Series #2003-54*, Board of Governors of the Federal Reserve System.
- [19] Gertler, M. (1999), “Government Debt and Social Security in a Life-Cycle Economy”, *Carnegie-Rochester Conference on Public Policy* 50, pp. 61-110.
- [20] Ghironi, F. (2003), “Macroeconomic Interdependence under Incomplete Markets”, *Working Paper*, Boston College.
- [21] Gourinchas, P.O. and H. Rey (2005), “International Financial Adjustment”, *National Bureau of Economic Research Working Paper #11155*.
- [22] Henriksen, E. (2002), “A Demographic Explanation of U.S. and Japanese Current Account Behavior”, *Working Paper*, University of Oslo.
- [23] Hurd, M. (1989), “Mortality Risk and Bequests”, *Econometrica* 57, pp. 779-813.
- [24] Juillard, M. (1996), “DYNARE: A program for the resolution and simulation of dynamic models with forward variables through the use of a relaxation algorithm”, *Couverture Orange #9602*, CEPREMAP.
- [25] Kreps, D. and E. Porteus (1978), “Temporal Resolution of Uncertainty and Dynamic Choice Theory”, *Econometrica* 46, pp. 185-200.

- [26] Lane, P. and G.M. Milesi-Ferretti (2001), “Long Term Capital Movements”, in *National Bureau of Economic Research Macroeconomics Annual*, B. Bernanke and K. Rogoff (eds.), MIT Press – Cambridge, MA.
- [27] Obstfeld, M. and K. Rogoff (1996), *Foundations of International Macroeconomics*, The MIT Press – Cambridge, MA.
- [28] Schmitt-Grohé, S. and M. Uribe (2003), “Closing Small Open Economy Models”, *Journal of International Economics* 61, pp. 163-185.
- [29] Weil, P. (1989), “Overlapping Families of Infinitely-Lived Agents”, *Journal of Public Economics* 38, pp.183-198.
- [30] Yaari, M. (1965), “Uncertain Lifetime, Life Insurance and the Theory of the Consumer”, *Review of Economic Studies* 32, pp. 137-150.

A Analytical Appendix

The first appendix describes the solution of the individual maximization problem and the characterization of the steady state.

A.1 The Individual Problem

The first order condition with respect to asset accumulation for a retiree born in period j and retired in period k yields

$$\left(C_t^{jkr}\right)^{\rho-1} = \beta\gamma_{t,t+1} \left(V_{t+1}^{jkr}\right)^{\rho-1} \frac{\partial V_{t+1}^{jkr}}{\partial A_{t+1}^{jkr}}.$$

The Envelope condition is

$$\frac{\partial V_t^{jkr}}{\partial A_t^{jkr}} = \left(V_t^{jkr}\right)^{1-\rho} \left(C_t^{jkr}\right)^{\rho-1} \frac{R_{W,t}}{\gamma_{t-1,t}}.$$

The resulting Euler equation takes the standard form

$$C_{t+1}^{jkr} = (\beta R_{W,t+1})^\sigma C_t^{jkr}, \quad (41)$$

where $\sigma \equiv (1 - \rho)^{-1}$. I can solve for the policy function of a retiree by guessing that consumption is a fraction of total wealth

$$C_t^{jkr} = \xi_t \left(\frac{R_{W,t} A_t^{jkr}}{\gamma_{t-1,t}} \right), \quad (42)$$

where ξ_t is the marginal propensity to consume for a retiree. If I substitute [42] into [41], I obtain

$$\xi_{t+1} \left(\frac{R_{W,t+1} A_{t+1}^{jkr}}{\gamma_{t,t+1}} \right) = (\beta R_{W,t+1})^\sigma \xi_t \left(\frac{R_{W,t} A_t^{jkr}}{\gamma_{t-1,t}} \right).$$

On the other hand, from the budget constraint of a retiree, one can substitute for consumption from [42] and express assets as

$$A_{t+1}^{jkr} = (1 - \xi_t) \frac{R_{W,t} A_t^{jkr}}{\gamma_{t-1,t}}.$$

I can then rewrite the last equation as

$$\frac{R_{W,t+1}A_{t+1}^{jkr}}{\gamma_{t,t+1}} = (1 - \xi_t) \frac{R_{W,t+1}}{\gamma_{t,t+1}} \left(\frac{R_{W,t}A_t^{jkr}}{\gamma_{t-1,t}} \right)$$

If I plug back the result into the Euler equation, it follows that the marginal propensity to consume follows a non-linear first order difference equation of the form

$$\xi_t = 1 - \gamma_{t,t+1} \beta^\sigma R_{W,t+1}^{\sigma-1} \frac{\xi_t}{\xi_{t+1}}. \quad (43)$$

I can also conjecture that the value function is linear in consumption according to

$$V_t^{jkr} = \Delta_t^r C_t^{jkr}. \quad (44)$$

Then, it must be the case that

$$\left(\Delta_t^r C_t^{jkr} \right)^\rho = \left(C_t^{jkr} \right)^\rho + \beta \gamma_{t,t+1} \left(\Delta_{t+1}^r C_{t+1}^{jkr} \right)^\rho.$$

I substitute for consumption at $t+1$ from [41] and simplify the consumption term. The result is

$$\left(\Delta_t^r \right)^\rho = 1 + \gamma_{t,t+1} \beta^\sigma R_{W,t+1}^{\sigma-1} \left(\Delta_{t+1}^r \right)^\rho.$$

From [43], it then follows that the proportionality term is

$$\Delta_t^r = \xi_t^{-\frac{1}{\rho}}.$$

The problem of a worker born in period j who is employed in period t is slightly more complicated but can be solved along the same lines. The first order condition for asset holding is

$$\left(C_t^{jw} \right)^{\rho-1} = \beta \left[\omega V_{t+1}^{jw} + (1 - \omega) V_{t+1}^{jr} \right]^{\rho-1} \left[\omega \frac{\partial V_{t+1}^{jw}}{\partial A_{t+1}^{jw}} + (1 - \omega) \frac{\partial V_{t+1}^{jr}}{\partial A_{t+1}^{jw}} \right].$$

The envelope conditions are

$$\frac{\partial V_t^{jw}}{\partial A_t^{jw}} = \left(V_t^{jw} \right)^{1-\rho} \left(C_t^{jw} \right)^{\rho-1} R_{W,t}$$

and

$$\frac{\partial V_t^{jr}}{\partial A_t^{jw}} = \frac{\partial V_t^{jr}}{\partial A_t^{jr}} \frac{\partial A_t^{jr}}{\partial A_t^{jw}} = \frac{\partial V_t^{jr}}{\partial A_t^{jr}},$$

where the last result is driven by the fact that individuals are risk neutral with respect to labor income fluctuations and hence choose the same asset profile independently of the employment status. I can then use the Envelope condition for retirees with the further adjustment that workers do not turn their wealth over to the mutual fund and hence do not receive the additional return that compensates for the probability of death. The Euler equation becomes

$$\begin{aligned} \left(C_t^{jw}\right)^{\rho-1} &= \beta R_{W,t+1} \left[\omega V_{t+1}^{jw} + (1-\omega) V_{t+1}^{jr} \right]^{\rho-1} \\ &\quad \left[\omega \left(V_{t+1}^{jw}\right)^{1-\rho} \left(C_{t+1}^{jw}\right)^{\rho-1} + (1-\omega) \left(V_{t+1}^{jr}\right)^{1-\rho} \left(C_{t+1}^{jr}\right)^{\rho-1} \right] \end{aligned} \quad (45)$$

I conjecture that the value function for a worker has the same form of [44]

$$V_t^{jw} = \Delta_t^w C_t^{jw}. \quad (46)$$

I substitute the guess back into the Euler equation [45] together with [44]

$$\left(C_t^{jw}\right)^{\rho-1} = \beta R_{W,t+1} \left[\omega \Delta_{t+1}^w C_{t+1}^{jw} + (1-\omega) \Delta_{t+1}^r C_{t+1}^{jr} \right]^{\rho-1} \left[\omega \left(\Delta_{t+1}^w\right)^{1-\rho} + (1-\omega) \left(\Delta_{t+1}^r\right)^{1-\rho} \right].$$

I further define the ratio of the quantity

$$\Omega_t \equiv \omega + (1-\omega) \left(\frac{\Delta_t^r}{\Delta_t^w} \right)^{1-\rho}.$$

The Euler equation can then be rearranged to give

$$\omega C_{t+1}^{jw} + (1-\omega) \left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w} \right) C_{t+1}^{jr} = (\beta \Omega_{t+1} R_{W,t+1})^\sigma C_t^{jw}. \quad (47)$$

I guess that the decision rule for a worker is

$$C_t^{jw} = \pi_t \left(R_{W,t} A_t^{jw} + H_t^{jw} \right). \quad (48)$$

From [42], I can also notice that the decision rule for a retiree who just abandoned the labor force is

$$C_t^{jr} = \xi_t R_{W,t} A_t^{jw}.$$

The variable H_t^{jw} represents the present discounted value of current and future human wealth and is defined in section 4. By substituting the guess into the Euler equation, I obtain

$$\omega \pi_{t+1} \left(R_{W,t+1} A_{t+1}^{jw} + H_{t+1}^{jw} \right) + (1 - \omega) \left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w} \right) \xi_{t+1} R_{W,t+1} A_{t+1}^{jw} = (\beta \Omega_{t+1} R_{W,t+1})^\sigma \pi_t \left(R_{W,t} A_t^{jw} + H_t^{jw} \right)$$

I can rearrange the last expression to yield

$$\omega \left(A_{t+1}^{jw} + \frac{H_{t+1}^{jw}}{R_{W,t+1}} \right) + (1 - \omega) \left(\frac{\Delta_{t+1}^r}{\Delta_{t+1}^w} \right) \epsilon_{t+1} A_{t+1}^{jw} = (\beta \Omega_{t+1})^\sigma R_{W,t+1}^{\sigma-1} \frac{\pi_t}{\pi_{t+1}} \left(R_{W,t} A_t^{jw} + H_t^{jw} \right),$$

where $\epsilon_t \equiv \xi_t / \pi_t$. I use the definition of Ω_t to rewrite

$$A_{t+1}^{jw} + \frac{\omega H_{t+1}^{jw}}{\Omega_{t+1} R_{W,t+1}} = \beta^\sigma (\Omega_{t+1} R_{W,t+1})^{\sigma-1} \frac{\pi_t}{\pi_{t+1}} \left(R_{W,t} A_t^{jw} + H_t^{jw} \right)$$

From the budget constraint of a worker and the guess [48], it is possible to see that

$$A_{t+1}^{jw} + \frac{\omega H_{t+1}^{jw}}{\Omega_{t+1} R_{W,t+1}} = (1 - \pi_t) \left(R_{W,t} A_t^{jw} + H_t^{jw} \right).$$

If I substitute this result back into the Euler equation, I can show that the marginal propensity to consume for a worker evolves according to

$$\pi_t = 1 - \beta^\sigma (\Omega_{t+1} R_{W,t+1})^{\sigma-1} \frac{\pi_t}{\pi_{t+1}} \quad (49)$$

Finally, I can verify the validity of the original guess for the value function [46]. The following expression must be satisfied

$$\left(\Delta_t^w C_t^{jw} \right)^\rho = \left(C_t^{jw} \right)^\rho + \beta \left[\omega \Delta_{t+1}^w C_{t+1}^{jw} + (1 - \omega) \Delta_{t+1}^r C_{t+1}^{jr} \right]^\rho.$$

I can use expression [47] to obtain

$$(\Delta_t^w)^\rho = 1 + \beta^\sigma (\Omega_{t+1} R_{W,t+1})^{\sigma-1} (\Delta_{t+1}^w)^\rho.$$

Expression [49] then implies that

$$\Delta_t^w = \pi_t^{-\frac{1}{\rho}}.$$

I can also observe that $\Delta_{t+1}^r/\Delta_{t+1}^w = \epsilon_t^{-(1/\rho)}$ and that

$$\Omega_t \equiv \omega + (1 - \omega) \epsilon_t^{\frac{1}{1-\sigma}}.$$

This concludes the characterization of the individual problem in the model.

A.2 The Steady State

In this section, I characterize the stationary symmetric steady state of the model. Symmetry implies that the stock of net foreign assets and the trade balance are zero.

For retirees and workers, it suffices to describe the steady state value of their respective marginal propensities to consume. For a given level of the world interest rate, the marginal propensity to consume of each cohort is given by

$$\xi = 1 - \gamma \beta^\sigma R_W^{\sigma-1} \quad (50)$$

and

$$\pi = 1 - \beta^\sigma (\Omega R_W)^{\sigma-1}, \quad (51)$$

where

$$\Omega = \omega + (1 - \omega) \epsilon^{\frac{1}{1-\sigma}} \quad (52)$$

and

$$\epsilon = \xi/\pi. \quad (53)$$

At this point, I prove two results related to the individual marginal propensities to consume.

First, I want to show that in steady state, $\epsilon > 1$. To see this, suppose the contrary is true ($\epsilon \leq 1$). Taking the ratio of [50] and [51] gives

$$\gamma \geq \Omega^{\sigma-1} = \left[\omega + (1 - \omega) \epsilon^{\frac{1}{1-\sigma}} \right]^{\sigma-1}.$$

Since $\gamma < 1$, the last condition is satisfied only if $\epsilon > 1$ which constitutes a contradiction. For a given level of the world interest rate, the system of equations [50]-[53] determine the solution of the individual marginal propensities to consume and for the variables Ω and ϵ .

Second, I show that in steady state, for a given interest rate, the probability of surviving has a negative effect on the marginal propensity to consume for workers. To see this, I can

rewrite expression [51] making use of [50], [52] and [53] as

$$\pi = 1 - \beta^\sigma \left\{ \left[\omega + (1 - \omega) \left(\frac{1 - \gamma \beta^\sigma R_W^{\sigma-1}}{\pi} \right)^{\frac{1}{1-\sigma}} \right] R_W \right\}^{\sigma-1}.$$

In order to sign the effect of γ on π (for a given R_W), I can just apply the implicit function theorem to the function

$$F(\gamma, \pi) = \pi - 1 + \beta^\sigma [\Omega(\gamma, \pi) R_W]^{\sigma-1}.$$

Some algebra allows to obtain the following partial derivatives

$$\frac{\partial F}{\partial \gamma} = (1 - \omega) \beta^{2\sigma} \Omega^{\sigma-2} R_W^{\sigma-1} \epsilon^{\frac{\sigma}{1-\sigma}} / \pi > 0$$

and

$$\frac{\partial F}{\partial \pi} = 1 + (1 - \omega) \beta^\sigma \Omega^{\sigma-2} R_W^{\sigma-1} \epsilon^{\frac{1}{1-\sigma}} / \pi > 0.$$

It then follows that

$$\frac{\partial \pi}{\partial \gamma} = - \frac{\partial F(\gamma, \pi) / \partial \gamma}{\partial F(\gamma, \pi) / \partial \pi} < 0,$$

which implies a negative effect of population aging on the marginal propensity to consume for workers, holding fixed the world interest rate. I now return to the description of the steady state equations.

From the production function it is easy to see that

$$y = k^{1-\alpha} \tag{54}$$

The firm first order condition for capital pins down the capital-output ratio for a given level of the world return

$$R_W = (1 - \alpha) \frac{y}{k} + (1 - \delta).$$

The last two expressions combined deliver a solution for capital per unit of efficient labor

$$k = \left[\frac{1 - \alpha}{R_W - (1 - \delta)} \right]^{\frac{1}{\alpha}}. \tag{55}$$

Investment is a constant share of the capital stock to compensate for depreciation and pop-

ulation growth

$$i = (x + n + \delta) k. \quad (56)$$

Again, for a given level of the world interest rate, the solution for capital, output and investment can be retrieved from the system of equations [54]-[56].

As a second step, I consider aggregate consumption and the distribution of assets. The equation for aggregate consumption [22] yields

$$c = \pi \{ [1 + (\epsilon - 1) \lambda] R_W a + h \}. \quad (57)$$

In steady state, the distribution of wealth across cohorts is determined by

$$\lambda = \frac{(1 - \omega)}{(1 + x + n) - \omega(1 - \xi) R_W}.$$

The expression for λ can also be readjusted as to make explicit the dependence on the ratio of retirees to workers in the population

$$\lambda = \frac{(1 + n - \gamma) \psi}{(1 + x + n) - \gamma \omega (\beta R_W)^\sigma}, \quad (58)$$

where

$$\psi = \frac{1 - \omega}{1 + n - \gamma}. \quad (59)$$

Given that the dependency ratio is only a function of exogenous parameters, the steady state distribution of wealth only depends on the level of the interest rate. The capitalized value of human wealth that enter the steady state consumption function is

$$h = \frac{(\alpha - \tau) y}{1 - \omega(1 + x) / \Omega R_W}. \quad (60)$$

In a symmetric steady state, net foreign assets are equal to zero so that total assets are given by

$$a = k + by. \quad (61)$$

From the government budget constraint, taxes equal expenditure plus the net interest rate payment on the stock of steady state debt

$$\tau = g + [R_W - (1 + x + n)] b. \quad (62)$$

The system of equations [57]-[62] determines the demand side of the model in steady state

for a given interest rate, which can be derived from the resource constraint

$$(1 - g)y = c + i, \tag{63}$$

given the steady state value of the stock of government debt b and of government spending g .

The algorithm of solution for the steady state corresponds to the one described in the text and the practical implementation is carried out using DYNARE.

B Data Appendix

In this appendix, I provide the details about the data used in this paper and the methodology to construct aggregates for the G6.

The series for the U.S. trade balance (versus the rest of the world and versus the G6) are from the Bureau of Economic Analysis (International Transactions) and are expressed in percentage of GDP (same source, NIPA Tables). Since bilateral trade data for France, Germany and Italy are available only starting in 1986, I proxy the G6 series using Western Europe plus Canada and Japan for the period 1960 – 1966 and the EU 6 (Belgium, France, Germany, Italy, Luxembourg and Netherlands) plus Canada, Japan and United Kingdom for the period 1966 – 1985. All series are expressed in current U.S. dollars. The trend for the U.S. trade balance vis-a-vis the G6 in percentage of GDP is the long run component of the Hodrick-Prescott filtered series.

The data used to construct real GDP growth rates (per capita and per worker) are from the Groningen Growth and Development Centre (GGCD). All values are expressed in PPP at 2002 U.S. dollars. Since total GDP is expressed in the same unit for all countries, I create a series for the G6 GDP by simply summing GDP across countries.

The data on historical and projected demographic trends are from United Nations World Population Prospects: The 2004 Revision. Although availability dates back to 1950, I use data and projections for the period 1970 – 2050. Population growth rates for the G6 are computed after aggregating total population across countries. The years of life expectancy, the dependency ratio and the fertility rate are a population-weighted average of the national counterparts.

The data for fiscal policy are from the International Monetary Fund (World Economic Outlook Database) and cover the period 1980 – 2004. I use PPP conversion factors from the

same source to express the levels of net government debt and total government deficit in each G6 member in 2000 U.S. dollars. I then aggregate across countries and divide by the series of GDP constructed along the same lines. The data for gross government debt used to extend the series for net debt back to 1970 are from the Organization for Economic Cooperation and Development (Economic Outlook Database). The aggregate for the G6 is constructed adopting the same methodology just described above.

Finally, the data used to construct the world interest rate are from the International Monetary Fund (International Financial Statistics) for the period 1980 – 2004. For each country, I compute the ex-post real interest rate by subtracting the realized CPI inflation rate from the available long term (generally 10-year) government bond yield. The “world” interest rate is computed using population weights³⁰.

³⁰Aggregation using GDP weights delivers almost identical results.