On the central role of Somers’ $D$

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This presentation can be downloaded from the conference website at
http://ideas.repec.org/s/boc/usug06.html
What is Somers’ $D$?
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  $$\tau_{XY} = E[\text{sign}(X_i - X_j)\text{sign}(Y_i - Y_j)]$$

  or as the difference between the probabilities of concordance and discordance between the two $(X, Y)$-pairs.
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or as the difference between the two corresponding conditional probabilities, given that one $X$–value is known to be larger than the other $X$–value.
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- **Somers’** $D$ is defined as the ratio

  $$D_{YX} = \tau_{XY}/\tau_{XX}$$

  or as the difference between the two corresponding *conditional* probabilities, given that one $X$–value is known to be larger than the other $X$–value.

- These definitions can be extended to cases where the $X$–values and/or the $Y$–values may be weighted and/or left–censored and/or right–censored.
You have already met Somers’ $D$
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- If $X$ and $Y$ are both binary, then Somers’ $D$ is the difference between proportions:

$$D_{YX} = \Pr(Y = 1|X = 1) - \Pr(Y = 1|X = 0)$$
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- Special cases include the population attributable risk, the ROC area, Harrell’s $c$ index, the Gini inequality index, and the parameters behind the “non–parametric” sign test and Wilcoxon and Gehan–Breslow ranksum tests.
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- However, $D_{YX}$ exists whether or not $X$ is binary, and is used to define...
Median differences and slopes
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- The **Theil–Sen median slope** of $Y$ with respect to $X$ is defined as a solution in $\beta$ to the equation

  \[ D_{Y-\beta X,X} = 0 \]

  or (in words) as a linear effect of $X$ on $Y$ sufficient to explain the observed Somers’ $D$. 
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• If $X$ is binary, then the Theil–Sen median slope is known as the Hodges–Lehmann median difference between groups $X = 1$ and $X = 0$. 
The Stata 9 version of the somersd package
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The `somersd` package, downloadable from SSC, has 3 modules to calculate confidence intervals for a large family of rank statistics:
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- The module *somersd* estimates Somers’ $D$, Harrell’s $c$ or Kendall’s $\tau_a$, saving the results as estimation results.
- The module *censlope* estimates Somers’ $D$, and then estimates the corresponding Theil–Sen median slope.
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All of these rank parameters have multiple versions for multiple sampling designs, with data weighted and/or censored and/or clustered and/or stratified.
Example: Prenatal paracetamol exposure and IgE
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- We will re–measure this association, using censlope to estimate Somers’ $D$ and Hodges–Lehmann median ratios.
Distribution of IgE in the 4848 children with IgE and paracetamol data
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- Total IgE, measured in kilounits/litre (kU/l), is raised in individuals with allergic diseases such as asthma.
- In the 4848 children with IgE and paracetamol data, its overall distribution is non-Normal.
- We wish to compare typical levels in the children of paracetamol users and non-users.
Comparing IgE levels using censlope
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- Given a randomly–chosen paracetamol–exposed child and a randomly–chosen paracetamol–unexposed child, Somers’ $D$ is the difference between the probability that the exposed child has the higher IgE and the probability that the unexposed child has the higher IgE.
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• (It is defined as the exponential of the Hodges–Lehmann median difference between the logged IgE values.)

• We will calculate confidence intervals for these two parameters, using censlope with Fisher’s $z$ transform.
. censlope lnigetot para32g, transf(z) eform;
Outcome variable: lnigetot
Somers’ D with variable: para32g
Transformation: Fisher’s z
Valid observations: 4848

Symmetric 95% CI for transformed Somers’ D

| para32g | Coef.  | Std. Err. | z     | P>|z|  | [95% Conf. Interval] |
|---------|--------|-----------|-------|------|----------------------|
| lnigetot | .0533954 | .0168421  | 3.17  | 0.002 | .0203856 -.0864053   |

Asymmetric 95% CI for untransformed Somers’ D

Somers_D  Minimum  Maximum
lnigetot  .05334475 .02038276 .0861909

95% CI(s) for percentile ratio(s)

<table>
<thead>
<tr>
<th>Percent</th>
<th>Pctl_Ratio</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.172549</td>
<td>1.0616111</td>
<td>1.2944986</td>
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</table>
How to adjust for confounders?
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- We fitted a logistic regression model to data from the 12127 children with data on maternal paracetamol use in late pregnancy.
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• We fitted a logistic regression model to data from the 12127 children with data on maternal paracetamol use in late pregnancy.

• Paracetamol exposure was regressed with respect to the following confounders: gender, maternal age, prenatal tobacco exposure, mother’s education, housing tenure, parity, maternal anxiety, maternal ethnic origin, multiple pregnancy, birth weight, gestational age at birth, head circumference, antibiotics in pregnancy, alcohol intake in pregnancy, maternal disease and infection history, younger siblings, presence of pets, breast feeding, day care, dampness problems, passive smoking exposure after birth, obesity index at 7 years.
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- The predicted log paracetamol odds, or propensity score, was grouped into 32 propensity strata, using `xtile`.
On the central role of Somers’ D

Paracetamol exposure prevalence in the 32 propensity groups

Propensity group for Paracetamol at 20–32 weeks gestation

Percent exposed to paracetamol

0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

Paracetamol exposure prevalence in the 32 propensity groups

Paracetamol propensity predicts paracetamol exposure, but not too well!
Within–strata rank statistics using \texttt{somersd}
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- Kendall’s $\tau_a$ and Somers’ $D$ can be restricted to comparisons within strata, using the `wstrata()` option of `somersd`.
Within-strata rank statistics using somersd

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• Therefore, so can median slopes, differences and ratios.
Within–strata rank statistics using \texttt{somersd}

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- \textit{Therefore}, so can median slopes, differences and ratios.
- We can therefore adjust our rank statistics for confounders by restricting to comparisons within the 32 propensity groups.
Within–strata rank statistics using `somersd`

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- Therefore, so can median slopes, differences and ratios.
- We can therefore adjust our rank statistics for confounders by restricting to comparisons within the 32 propensity groups.
- We will now estimate a propensity–adjusted Somers’ $D$ and median ratio, using `censlope`. 
On the central role of Somers’ D

```
censlope lnigetot para32g, transf(z) eform wstrata(pg_para32g);
Outcome variable: lnigetot
Somers’ D with variable: para32g
Transformation: Fisher’s z
Within strata defined by: pg_para32g
Valid observations: 4848

Symmetric 95% CI for transformed Somers’ D

<p>| | | | | |</p>
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<thead>
<tr>
<th></th>
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<tr>
<td>para32g</td>
<td>Coef.</td>
<td>Std. Err.</td>
<td>z</td>
<td>P&gt;</td>
</tr>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td>.0416191</td>
<td>.018089</td>
<td>2.30</td>
<td>0.021</td>
</tr>
</tbody>
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Asymmetric 95% CI for untransformed Somers’ D

Somers D Minimum Maximum
lnigetot .04159508 .00616518 .07692067

95% CI(s) for percentile ratio(s)

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<tr>
<td>50</td>
<td>1.1256541</td>
<td>1.0165742</td>
<td>1.2556066</td>
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- *However*, children in the same stratum have the same discrete propensity *group*, not the same continuous propensity *score*.

- *Therefore*, the association between paracetamol exposure and IgE within paracetamol propensity groups *might possibly* be due to a residual association of both variables with the paracetamol propensity score.
Is 32 propensity groups enough?

- 32 propensity groups is more than most statisticians use most of the time (typically 5).

- However, children in the same stratum have the same discrete propensity group, not the same continuous propensity score.

- Therefore, the association between paracetamol exposure and IgE within paracetamol propensity groups might possibly be due to a residual association of both variables with the paracetamol propensity score.

- Fortunately, somersd can help us to check this possibility.
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The two interpretations of Somers’ $D$

Given an outcome variable $Y$ and a predictor variable $X$, interpretations of Somers’ $D$ fall into two classes:

- We may interpret $D_{YX}$ as a measure of the **effect** of $X$ on $Y$, especially if $X$ is binary, as in the examples so far.

- Alternatively, we may interpret $D_{XY}$ as a **performance indicator** for $X$ as a predictor of $Y$, for comparison with another predictor $W$. 
The two interpretations of Somers’ $D$

Given an outcome variable $Y$ and a predictor variable $X$, interpretations of Somers’ $D$ fall into two classes:

- We may interpret $D_{YX}$ as a measure of the **effect** of $X$ on $Y$, especially if $X$ is binary, as in the examples so far.

- Alternatively, we may interpret $D_{XY}$ as a **performance indicator** for $X$ as a predictor of $Y$, for comparison with another predictor $W$.

The second interpretation is possible because, *if* a positive association of $Y$ with $X$ is caused entirely by a positive association of both variables with a third variable $W$, *then* we must have the inequality

$$D_{XY} \leq D_{WY}$$

(see Newson (2002) and Newson (2006)), and we can test this inequality using `somersd` and `lincom`. 
Comparing Somers’ $D$ parameters for paracetamol and paracetamol propensity
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Comparing Somers’ $D$ parameters for paracetamol and paracetamol propensity

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- We use `somersd` to estimate $D_{XY}$ and $D_{WY}$. 
Comparing Somers’ $D$ parameters for paracetamol and paracetamol propensity

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- We use `somersd` to estimate $D_{XY}$ and $D_{WY}$.

- Again, we use the options `wstrata(pg_para32g)` to compare children in the same propensity group, and `transf(z)` to use Fisher’s $z$–transform.
Comparing Somers’ $D$ parameters for paracetamol and paracetamol propensity

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- We use `somersd` to estimate $D_{XY}$ and $D_{WY}$.

- Again, we use the options `wstrata(pg_para32g)` to compare children in the same propensity group, and `transf(z)` to use Fisher’s $z$–transform.

- We then compare the $z$-transformed $D_{XY}$ and $D_{WY}$, using `lincom`.
On the central role of Somers’ D

```
.somersd lnigetot para32g ps_para32g, transf(z) wstrata(pg_para32g);
Somers’ D with variable: lnigetot
Transformation: Fisher’s z
Within strata defined by: pg_para32g
Valid observations: 4848

Symmetric 95% CI for transformed Somers’ D

|       |     Coef. | Std. Err. |    z  | P>|z| | [95% Conf. Interval] |
|-------|-----------|-----------|-------|------|----------------------|
| para32g | 0.0181683 | 0.0078918 | 2.30  | 0.021 | 0.0027006   0.033636 |
| ps_para32g | -0.0082111 | 0.0099832 | -0.82 | 0.411 | -0.0277777  0.0113556 |

Asymmetric 95% CI for untransformed Somers’ D

<table>
<thead>
<tr>
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<th>Minimum</th>
<th>Maximum</th>
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<tr>
<td>para32g</td>
<td>0.0181663</td>
<td>0.00270058</td>
<td>0.03362334</td>
</tr>
<tr>
<td>ps_para32g</td>
<td>-0.00821087</td>
<td>-0.0277706</td>
<td>0.01135515</td>
</tr>
</tbody>
</table>
```
Paracetamol exposure (\texttt{para32g}) is a significant positive predictor, and paracetamol propensity (\texttt{ps\_para32g}) is a non–significant negative predictor.
However, to test the inequality, we use \texttt{lincom} to define a confidence interval and a $P$–value for half the difference between the two $z$–transformed Somers’ $D$ parameters, as follows:
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```
   . lincom (para32g-ps_para32g)/2;
```

\[
(1) \ 0.5 \ para32g - 0.5 \ ps\_para32g = 0
\]

| lnigetot | Coef.   | Std. Err. | z      | P>|z| | [95% Conf. Interval] |
|----------|---------|-----------|--------|------|----------------------|
| (1)      | 0.01319 | 0.00636   | 2.07   | 0.038| 0.0007167 - 0.0256626 |


However, to test the inequality, we use `lincom` to define a confidence interval and a $P$–value for half the difference between the two $z$–transformed Somers’ $D$ parameters, as follows:

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( 1) .5 para32g - .5 ps_para32g = 0
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We see that the difference is (just) significantly positive. So the positive association between IgE and paracetamol exposure within paracetamol propensity groups is probably *not* due to a residual positive association of both variables with paracetamol propensity score.
IgE and prenatal paracetamol exposure: summary

Parameter type

Unstratified

Propensity-stratified

Median exposed/unexposed IgE ratio (95% CI)
IgE and prenatal paracetamol exposure: summary

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- If they are in the same paracetamol propensity group, then the exposed child typically has 2% to 26% more IgE.
- This relative difference is probably not caused by paracetamol propensity (as defined here).
The case for rank methods
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• *Also*, rank parameters are often easier to interpret (as differences between proportions, or as median differences or ratios).

• By contrast, an arithmetic mean difference is *usually* a proxy for a median difference, and *may* be expressed in incomprehensible units, such as a symptom score after a Normalizing transformation.
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- More people think that they cannot be adjusted for confounding variables.
- (They can, but we needed to use regression methods to define the propensity score.)
- A more valid argument is that of Fisher (1935), which implies that, if we know the distributional family a priori, then we can define narrower confidence intervals using maximum-likelihood methods than using rank methods.
- For instance, using a t-test instead of censlope may reduce the minimum detectable difference by a modest 5%, when comparing 2 samples of 40. Or from infinity to a finite difference, when comparing 2 samples of 3.
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• More work is needed (and is in progress) to find more quantitative information about these tradeoffs.
Summary

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• However, they are less robust to small sample numbers.

• More work is needed (and is in progress) to find more quantitative information about these tradeoffs.

• Meanwhile, I would like to thank StataCorp for the Mata programming language, which made `somersd` possible in its present form.
References


